Shear Stresses due to Forces

اجهادات القص نتيجة القوى

نسألكم الدعاء

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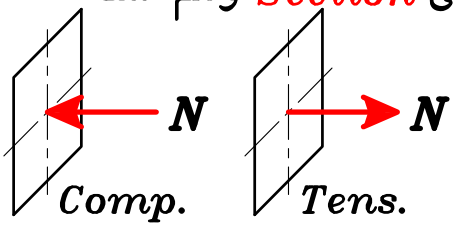
SHEAR STRESSES

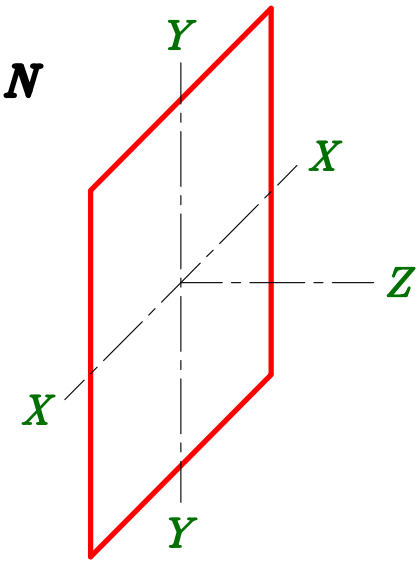
اجهادات القص

درسنا فى العام السابق الـ *Straining actions* وهى عبارة عن القوى التى من المحتمل أن تؤثر على أى *Section* وهم ستة *Straining actions*

1 – N (Normal Force)

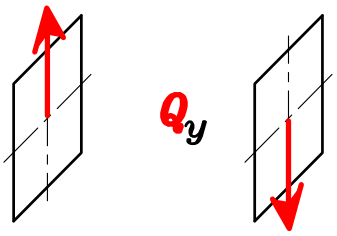
و هى القوة فى اتجاه محور (Z)
و تكون اما شد أو ضغط





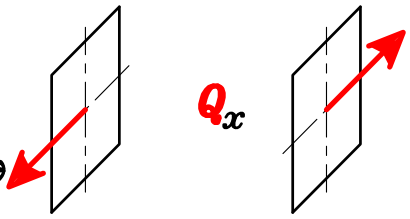
2 – Q_y (Shear Force)

و هى القوة فى اتجاه محور (Y)



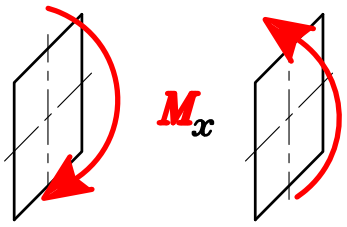
3 – Q_x (Shear Force)

و هى القوة فى اتجاه محور (X)



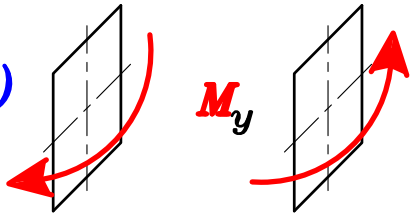
4 – M_x (Bending moment)

وهو عزم الدوران حول محور (X)



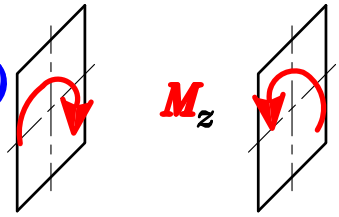
5 – M_y (Bending moment)

وهو عزم الدوران حول محور (Y)



6 – M_z (Torsional moment)

وهو عزم الدوران حول محور (Z)



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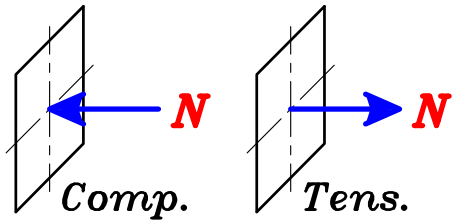
و الستة *Straining actions* ينتج عنهم نوعين من الاجهادات و هما

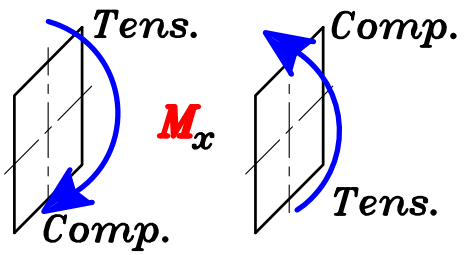
1 – Normal Stresses

و هى الـ *Stresses* التى ينتج عنها ضغط أو شد على الـ *Section* و قد درسنا هذه الـ *Stresses* فى العام السابق و الـ *Straining actions* التى ينتج عنها هذا النوع هم ثلاثة

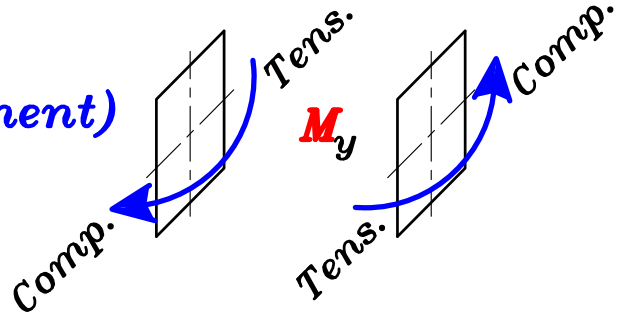
1 – **N (Normal Force)**



2 – **M_x (Bending moment)**



3 – **M_y (Bending moment)**

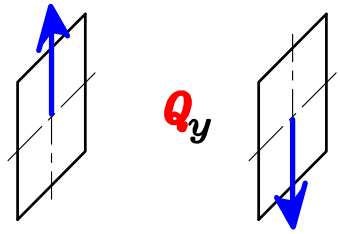


حيث أننا تعلمنا أن الـ *Compression* يكون ناحية رأس سهم الـ *moment* و الـ *Tension* يكون ناحية ذيل السهم .

2 – Shear Stresses

و هى الـ *Stresses* التى ينتج عنها *Friction* مع الـ *Section* و هذه هى موضوع هذا الدرس و الـ *Straining actions* التى ينتج عنها هذا النوع هم ثلاثة

1 – **Q_y (Shear Force)**



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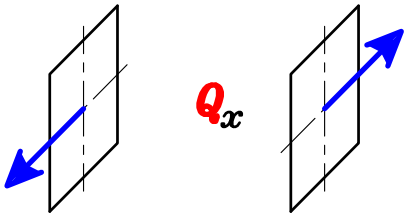
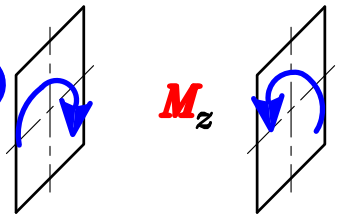
2 – Q_x (Shear Force)



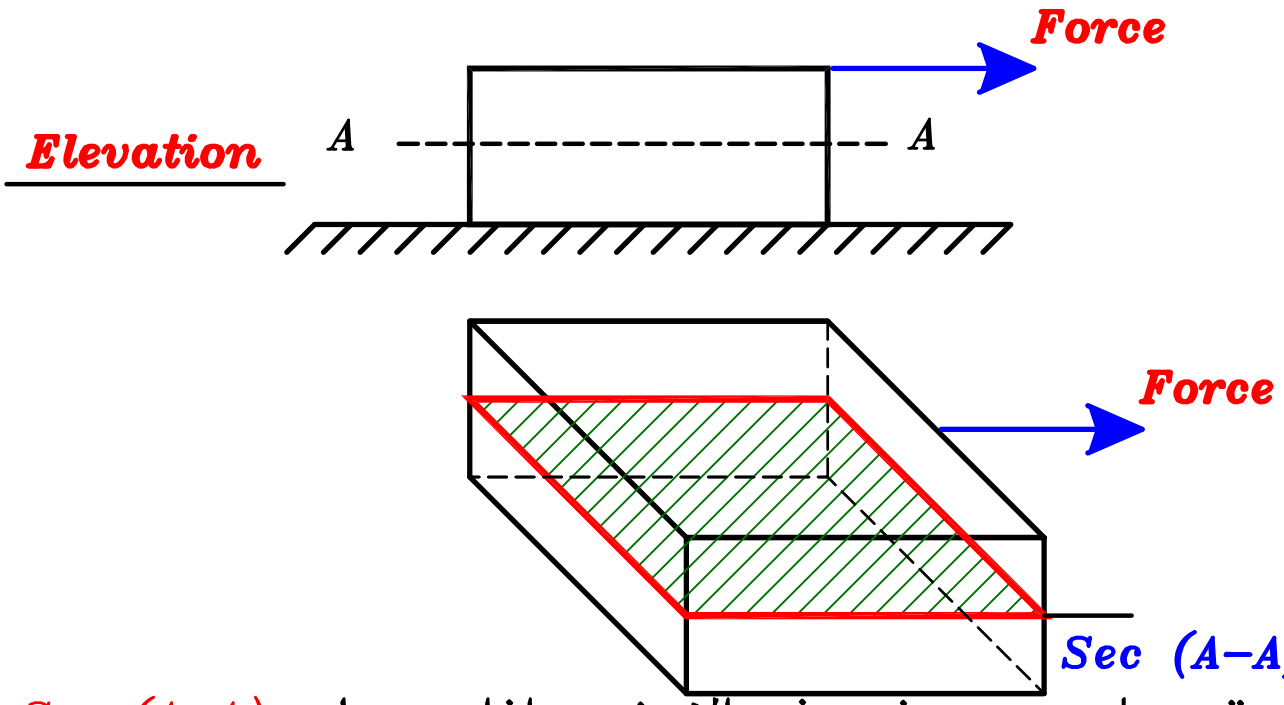
3 – M_z (Torsional moment)



و الـ Shear Stresses نحتاج اليها فى تطبيقات كثيرة مثل تصميم الكمرات و هذا ما سوف ندرسه فى مادة الخرسانة و أيضا عند تصميم قواعد المبانى و هذا و هذا ما سوف ندرسه فى مادة الـ Soil .

و لكى نتخيل الموضوع نأخذ مثال بسيط

SHEAR STRESSES DUE TO FORCES



اذا اثرت قوه على جسم مثبت فى الارض و اذا درسنا Sec (A–A) نجد ان النصف العلوى منه يريد ان يتحرك و النصف السفلى ثابت فى الارض فينشأ عند Sec (A–A) اجهادات لمقاومة الحركة فى الارض تسمى **SHEAR STRESSES**

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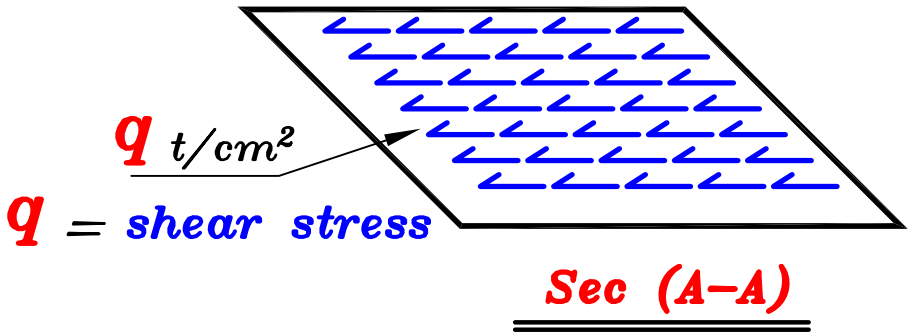
و أى اجهاد تكون وحداته عبارة عن وحدات(Force)على(Area)

m^2 ton

cm^2 KN

Kg



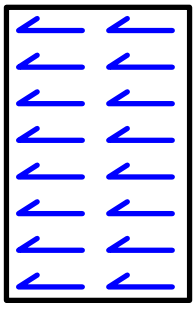
Sec (A-A)

و القانون العام لحساب الـ SHEAR STRESSES :

$$q = \frac{Q}{I} \; x \; \frac{S}{b}$$

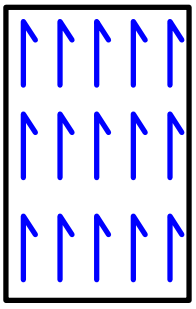
Where:

q = Shear stress



أفقى

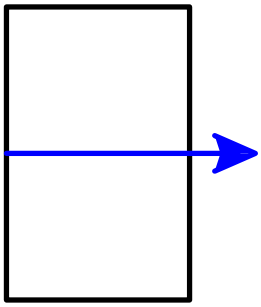
q_x



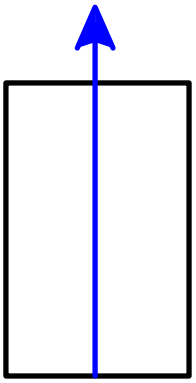
رأسى

q_y

Q = Shear Force



Q_x



Q_y

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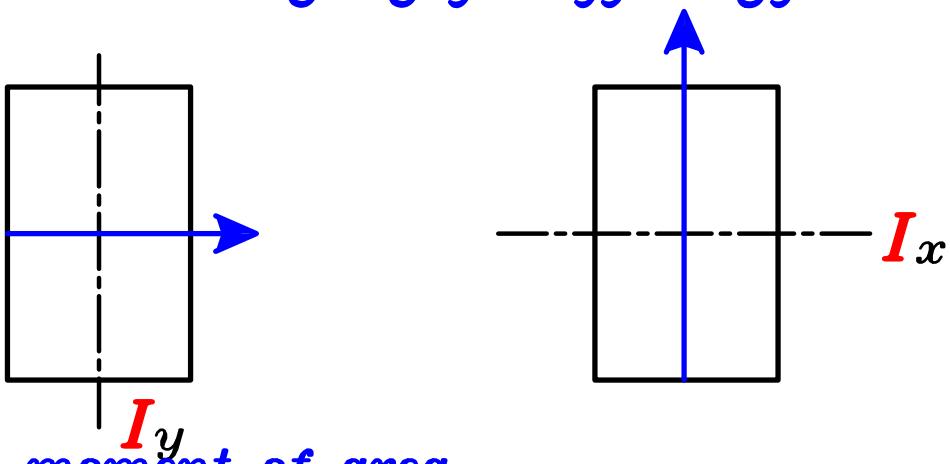
I = moment of inertia و هو عبارة عن عزم القصور الذاتى و تم دراسته فى العام السابق و توجد قيم للاشكال المحفوظة فى نهاية هذه الملزمة

و نحسب الـ I حول المحور العمودى على الـ Shear Force

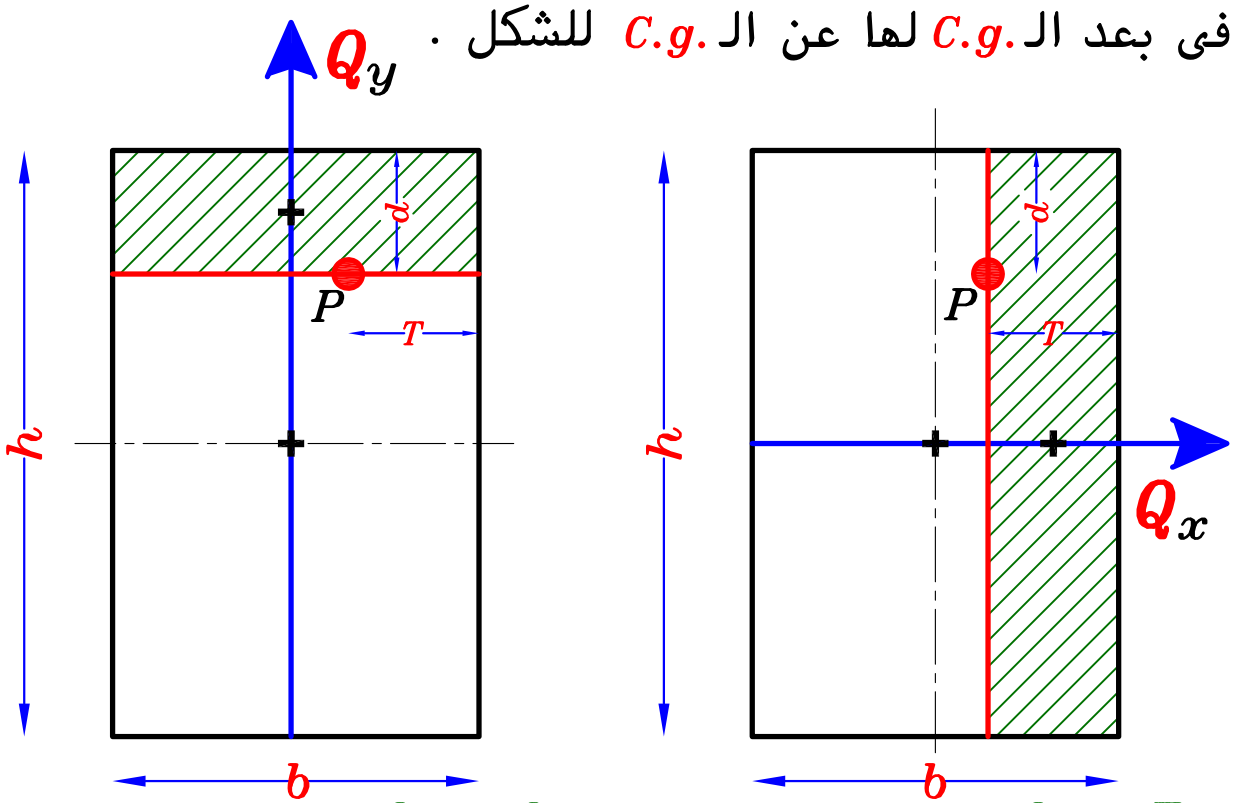


S = Statical moment of area

و تحسب عند النقطة المراد حساب الـ Shear Stresses عندها حول خط عمودى على الـ Shear Force عند هذه النقطة .

و لحساب الـ (S)

يتم عمل خط عمودى على (Q) من النقطة المراد حساب الـ Stresses عندها فتقسم الشكل الى مساحتين و تكون (S) هى احدى المساحتين مضروب فى بعد الـ C.g. لها عن الـ C.g. للشكل .



$$S = (b \times d)\left(\frac{h}{2} - \frac{d}{2}\right) \qquad S = (h \times T)\left(\frac{b}{2} - \frac{T}{2}\right)$$

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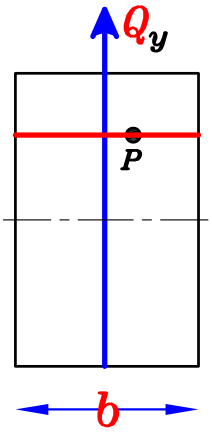
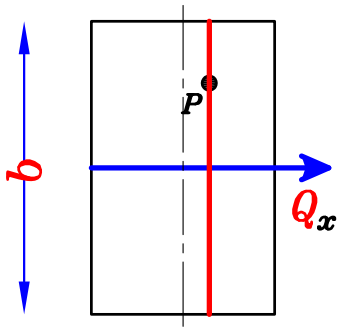
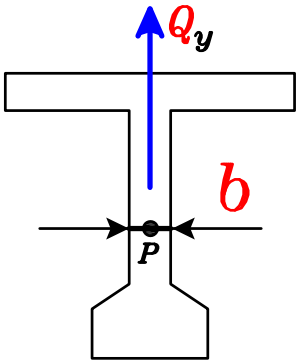
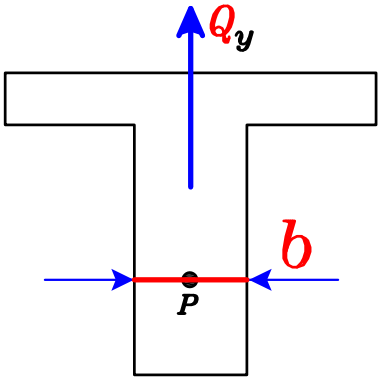
b = ***Shear Force*** عرض الجزء المصمت من ال ***Section*** عمودى على

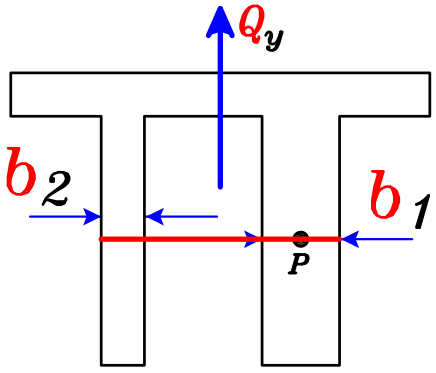
عند النقطة المراد عندها حساب ال ***Shear Stresses*** .



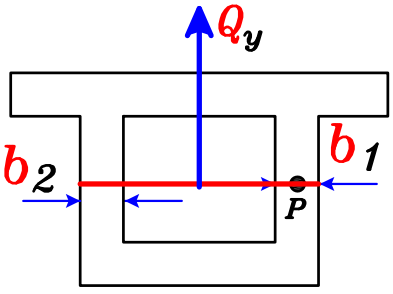




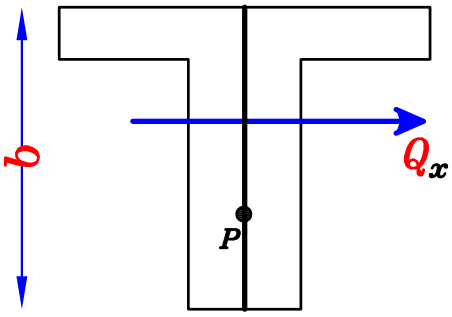


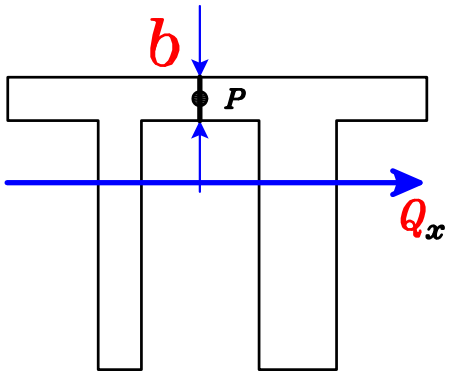


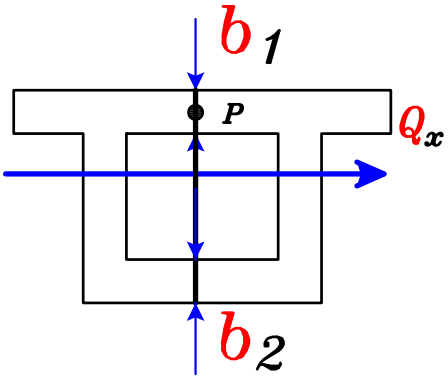
$b = b_1 + b_2$



$b = b_1 + b_2$







$b = b_1 + b_2$

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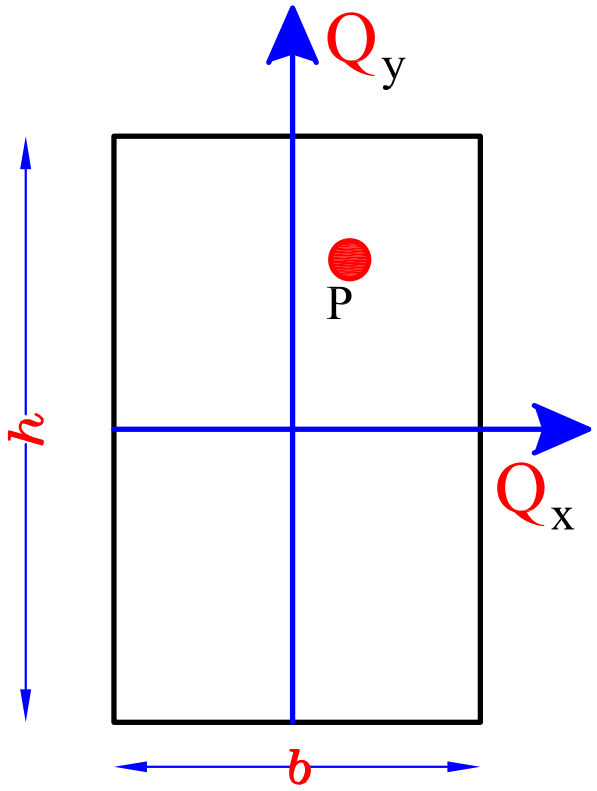
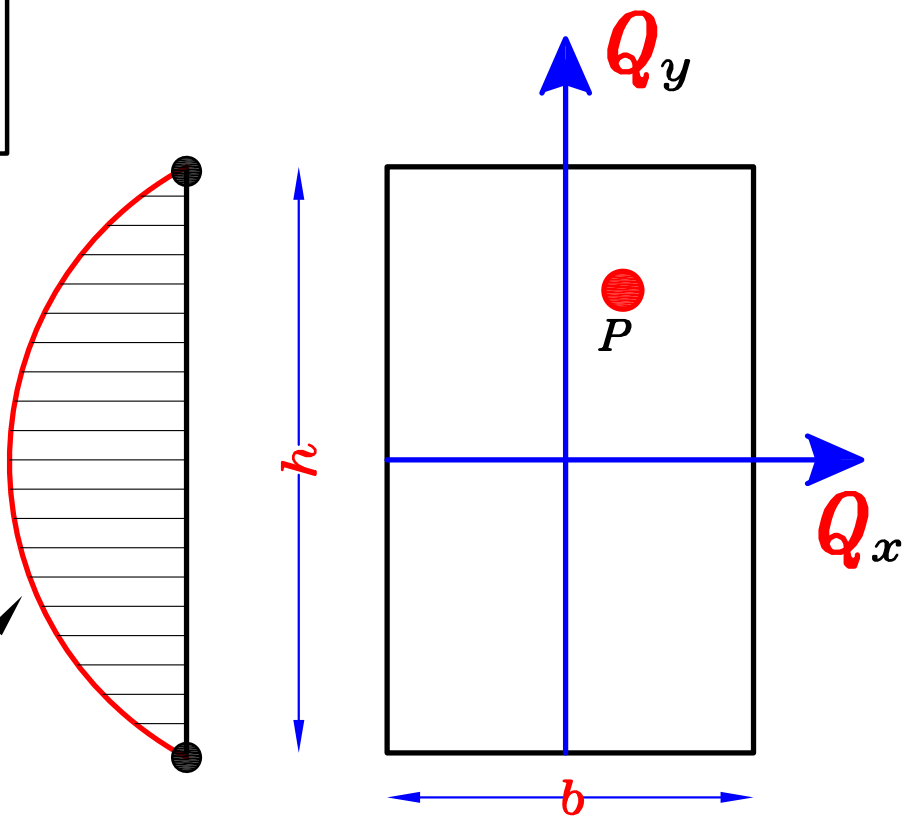
و لحساب الـ Shear Stresses عند نقطة (P) مثلا فى هذا الـ Section



لحساب الـ Shear Stress نتيجة Q_y

$$q_y = \frac{Q_y}{I_x} \; x \; \frac{S_x}{b}$$



توزيع الـ (Shear Stress)
على القطاع الناتج
من Q_y

و هذا سوف نتعلم رسمه

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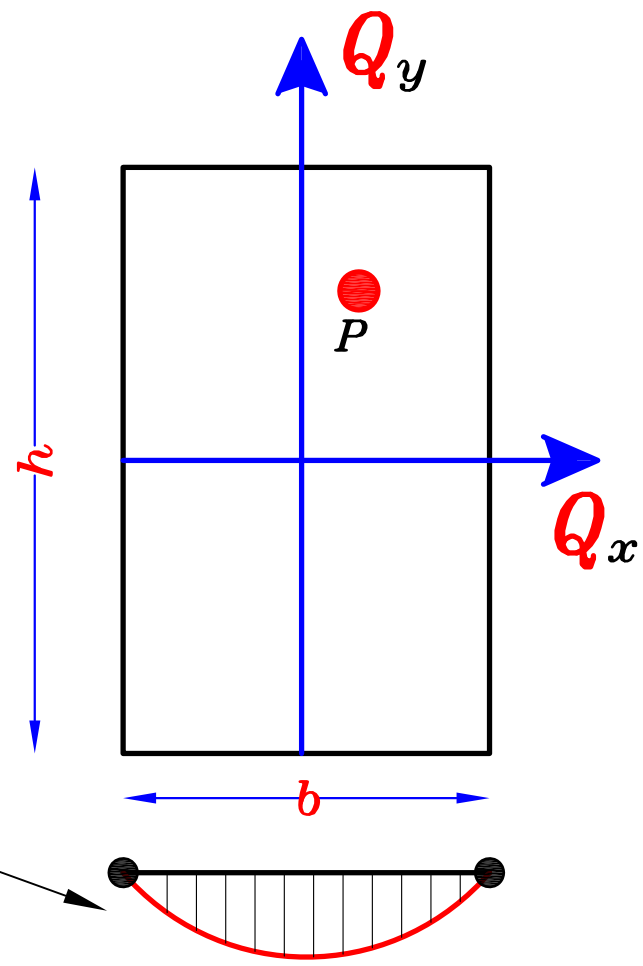
لحساب الـ Shear Stress نتيجة Q_x

$$q_x = \frac{Q_x}{I_y} \; x \; \frac{S_y}{b}$$

ملحوظة هامة

يرسم توزيع الـ (Shear Stress)

على خط موازى للـ Shear Force



توزيع الـ (Shear Stress)

على القطاع الناتج

من Q_x

و هذا سوف نتعلم رسمه

لحساب الـ maximum Shear Stresses

طبعا من أكثر ما يهمنا أن نعرف قيمة الـ maximum Shear Stresses و ذلك لاننا سوف نحتاج اليها فى التصميم كما سنأخذ فى الخرسانة و فى الـ Steel .

و بالنظر الى القانون $q = \frac{Q}{I} \; x \; \frac{S}{b}$

نجد أنه عند أى نقطة فى الـ Section لا تختلف قيمة الـ Q أو الـ I و لكن الذى يتغير هو قيمة الـ $\frac{S}{b}$ و بالتالى فان الـ maximum Shear Stresses يكون موجود عند النقطة التى عندها الـ $\frac{S}{b}$ max. .

$$\frac{Q}{I} = Constant.$$

$q \; \alpha \; \frac{S}{b}$ ⟶ $\frac{S}{b}$ max. عند تكون $(q \; max.)$

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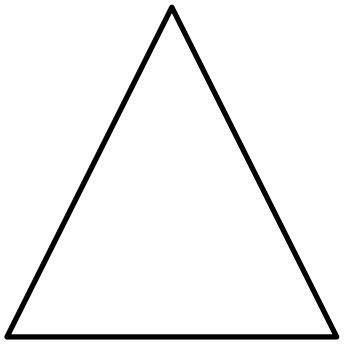
للحصول على maximum Shear Stresses

$$q = \frac{Q}{I} \times \frac{S}{b}$$

يوجد ٣ أنواع من الـ Sections

العرض متغير تدريجيا



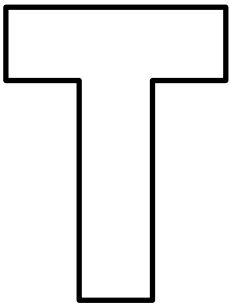
الـ $Q\&I$ ثوابت

∴ $q \ \alpha \ \frac{S}{b}$

∴ $q_{max.}$ is at $\left(\frac{S}{b}\right)_{max.}$

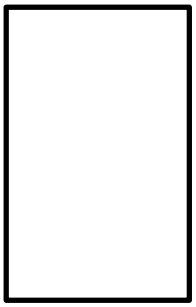
و لمعرفة مكان الـ $\left(\frac{S}{b}\right)_{max.}$ نحتاج الى عمل تفاضل للـ $\frac{S}{b}$ فى الاتجاه المراد حساب الـ Stress فيه

العرض متغير عند نقط معينة



سيتم حسابه فى المسائل

العرض ثابت



الـ $Q\&I\&b$ ثوابت

∴ $q \ \alpha \ S$

∴ $q_{max.}$ is at $S_{max.}$

∴ $S_{max.}$ is at $C.g$

∴ $q_{max.}$ is at $C.g$

١ - القطاعات ذات العرض الثابت (المستطيل أو المربع)

اثبات

For Q_y

$S_{max.}$ موجودة عند الـ $C.g$ فى حالة المستطيل و بالتالى $q_{max.}$ الـ عند الـ $C.g$

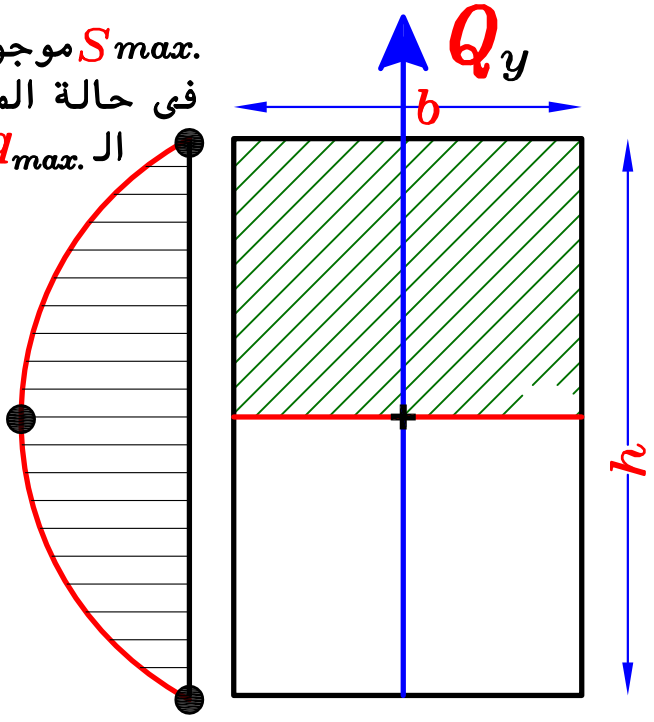
$$q_y = \frac{Q_y}{I_x} \times \frac{S_x}{b}$$

$$I_x = \frac{bh^3}{12}$$

$q_{max.}$ is at $C.g$

$S = A \times Y$

$$q_{max.} = 1.5 \frac{Q}{A}$$



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$$(S)\ at\ c.g. = (bxh/2)\ x\ (h/4) = bh^2/8$$

$$q_{max.} = \frac{Q_y}{bh^3/12}\ x\ \frac{bh^2/8}{b} = \frac{3}{2} x\ \frac{Q_y}{A}$$

$$\boxed{q_{max.} = 1.5\ \frac{Q_y}{A}}$$

يمكن استخدام هذا القانون مباشرة
عند حل المسائل

ملحوظة هامة

دائما قيمة الShear stress فى بداية و نهاية أى قطاع تساوى صفر و ذلك
لان الS تساوى صفر فى البداية أو النهاية

For Q_x

اثبات

$$q_x = \frac{Q_x}{I_y}\ x\ \frac{S_y}{b}$$

$$I_y = \frac{hb^3}{12}$$

$S_{max.}$ موجودة عند الC.g
فى حالة المستطيل و بالتالى
$q_{max.}$ عند الC.g

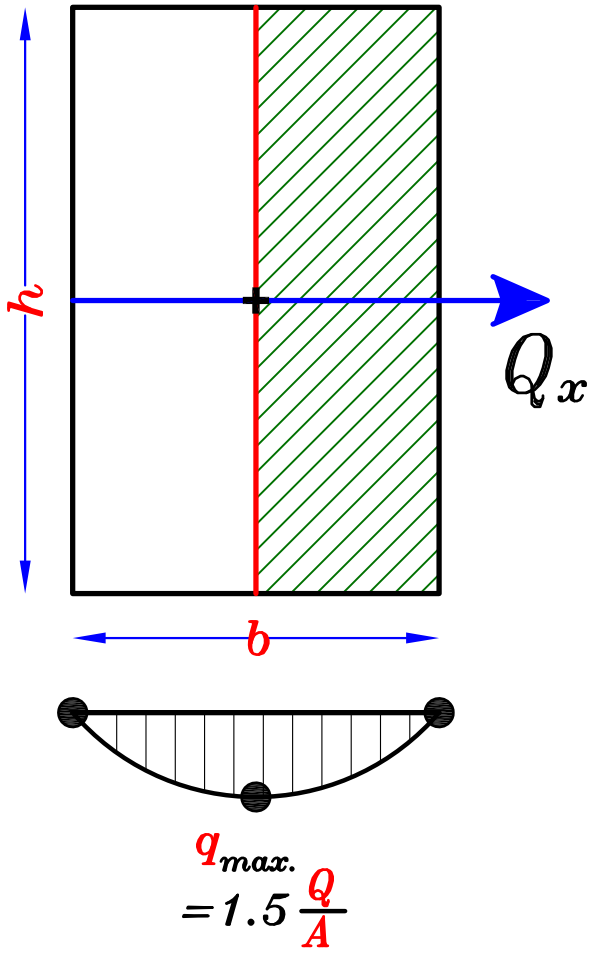
$q_{max.}$ is at C.g

$$S = A\ x\ Y$$

$$(S)\ at\ c.g.$$

$$= (hxb/2)\ x\ (b/4) = hb^2/8$$

$$q_{max.} = \frac{Q_y}{hb^3/12}\ x\ \frac{hb^2/8}{b} = \frac{3}{2} x\ \frac{Q_y}{A}$$



$$q_{max.} = 1.5\ \frac{Q}{A}$$

$$\boxed{q_{max.} = 1.5\ \frac{Q_y}{A}}$$

يمكن استخدام هذا القانون مباشرة
عند حل المسائل

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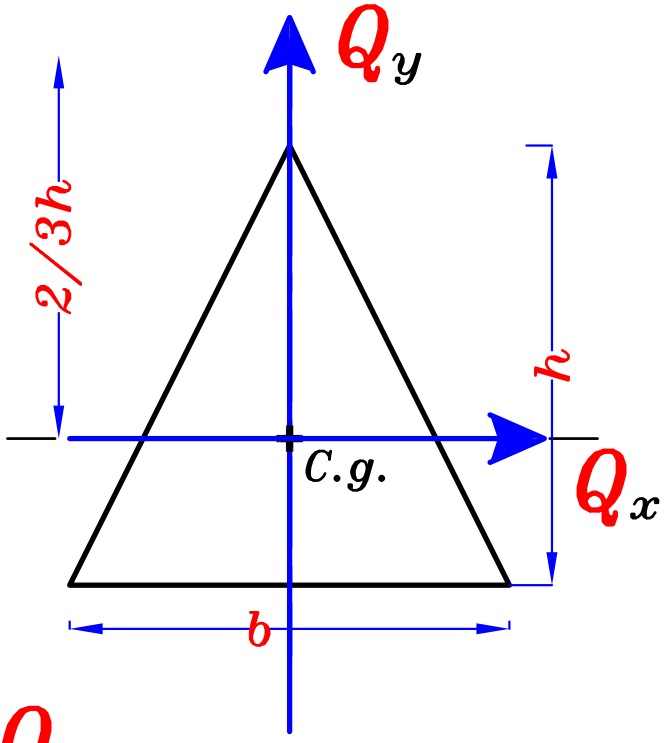
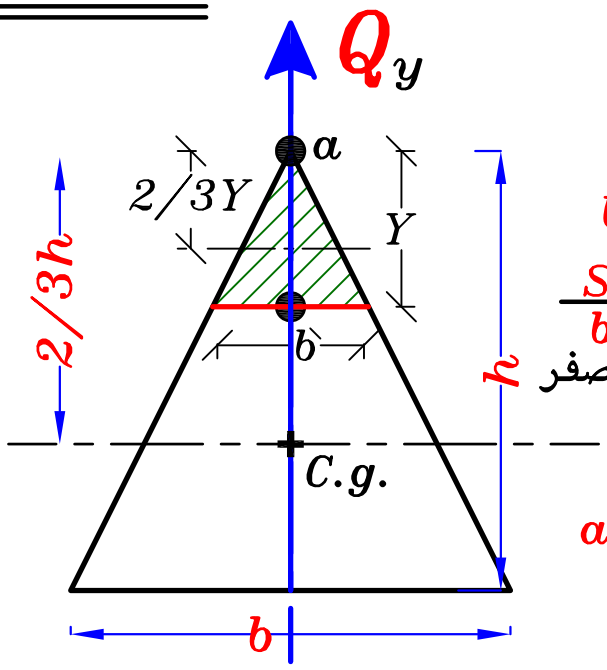
اثبات



For Qy



حيث ان العرض متغير نفرض نقطة على بعد y من نقطة a و عرض القطاع عندها $b`$

و يتم حساب $\frac{S}{b}$ عند هذه النقطة و نفاضل $\frac{S}{b}$ بالنسبة لاتجاه الـ *Shear Force* و نساوى بالصفر لايجاد قيمة y التى تجعل $\frac{S}{b}_{max.}$ ومنها نحصل على البعد الذى عنده $q_{max.}$ من نقطة a

$$S = [(1/2) (y) (b`)] [(2/3\ h) - (2/3\ y)]$$

$$S = [(1/2) (y) (b`)] [(2/3) (h-y)]$$

$$S = [(1/3) (b`) (hy - y^2)]$$

$$S/b = (1/3) (hy - y^2)$$

$$d(S/b)/dy = (1/3) (h - 2y)$$

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$d(S/b)/dy = zero \Longrightarrow$ **$y = h/2$**

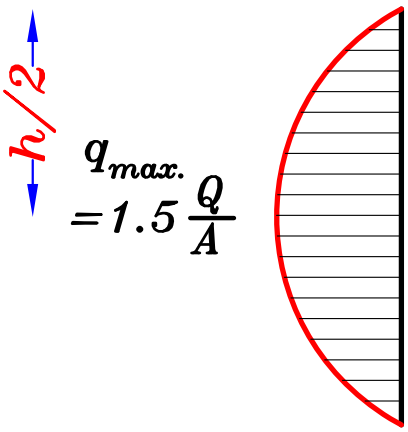
S/b max. @ $y = h/2$

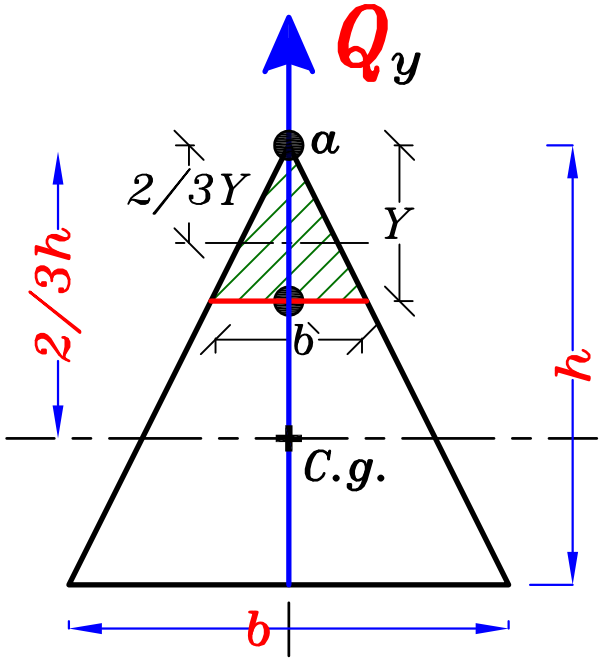
S/b max. $= (1 / 12)\ (h^2)$

$$I_x = \frac{bh^3}{36}$$

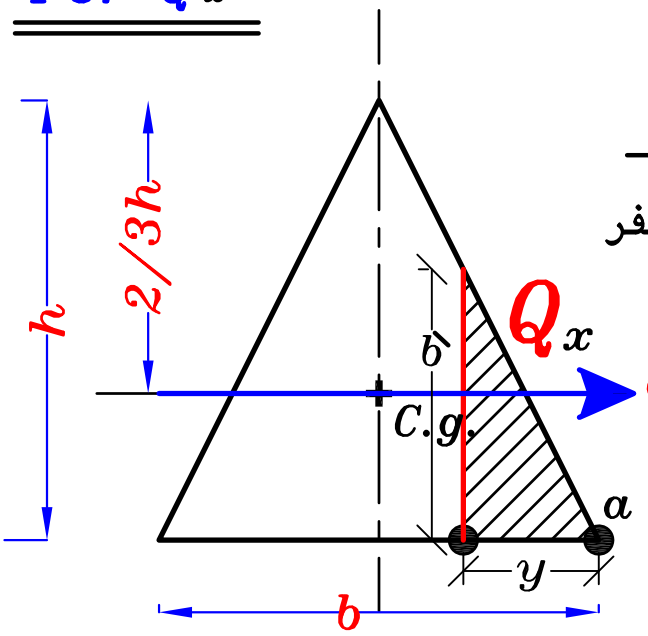
$$q_{max.} = \frac{Q_y}{bh^3/36} \; x \; \frac{h^2}{12} = \frac{3}{2} x \frac{Q_y}{A}$$

$$q_{max.} = 1.5 \frac{Q_y}{A}$$





For Q_x



حيث ان العرض متغير نفرض نقطة على بعد y من نقطة a و عرض القطاع عندها $b`$

و يتم حساب $\frac{S}{b}$ عند هذه النقطة و نفاضل $\frac{S}{b}$ بالنسبة لاتجاه الـ *Shear Force* و نساوى بالصفر لايجاد قيمة y التى تجعل $\frac{S}{b}_{max.}$ ومنها نحصل على البعد الذى عنده $q_{max.}$ من نقطة a

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$S = (1/2)\ (yb)\ (b/2 - 2y/3)$

$(S/b) = (1/2)\ (by/2 - 2/3\ y^2)$

$d(S/b)/dy = (1/2)\ (b/2 - 4/3\ y)$

$d(S/b)/dy = zero$ $\qquad \boxed{y = 3/8\ b}$

$S/b\ max.\ @\ y = 3/8\ b$

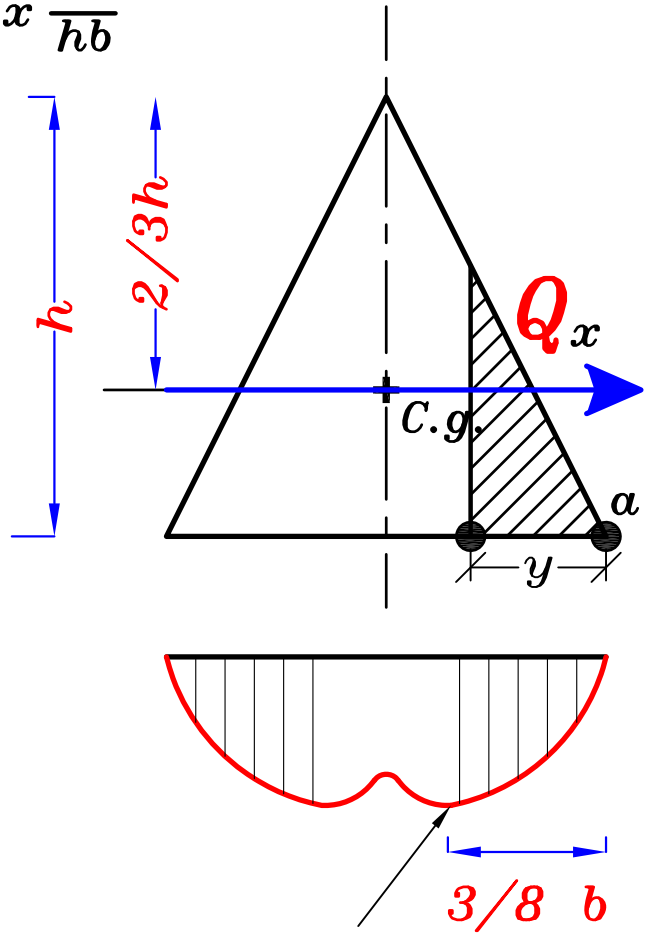
$$S/b\ max. = \left[\frac{3b^2}{8x2} - \frac{2\ x\ 9b^2}{3x64}\right]\ x\ (1/2)$$

$$= (1/2)\ (0.1875\ b^2 - 0.09375\ b^2) = 0.046875\ b^2$$

$$I_y = \frac{hb^3}{48}$$

$$q_{max.} = \frac{Q_x}{hb^3/48}\ x\ \frac{3b^2}{64} = \frac{9}{4}x\ \frac{Q_x}{hb}$$

$$\boxed{q_{max.} = \frac{9}{8}x\ \frac{Q_x}{A}}$$



$$q_{max.} = \frac{9}{8}x\ \frac{Q_x}{A}$$

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Circle

اثبات

$q_{max.}$ is at C.g

$$q_{max.} = \frac{Q}{I} x \left[\frac{S}{b}\right]_{C.g}$$

$$I = \frac{\pi R^4}{4}$$

$$S = (1/2)(\pi R^2)(4R/3\pi)$$

$$= (4/6) R^3$$

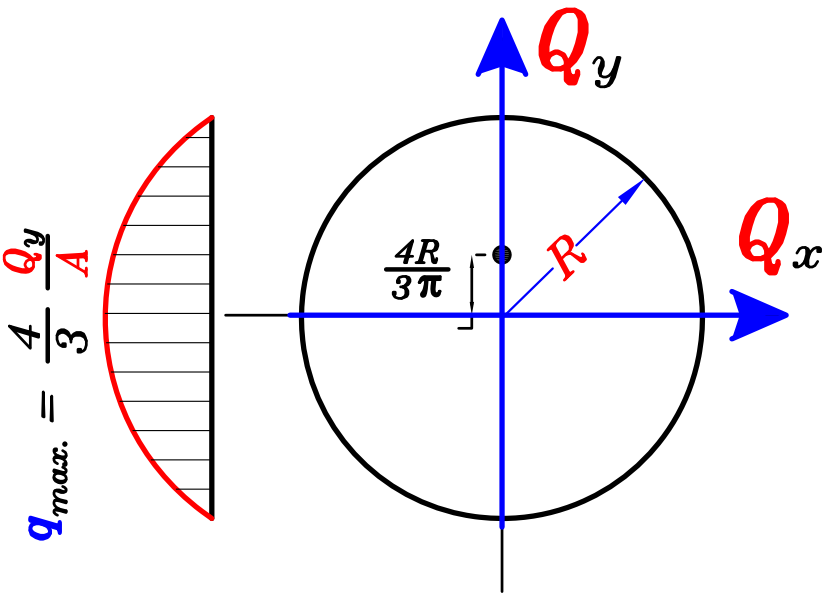
$$\frac{S}{b} = \frac{(4/6)R^3}{2R}$$

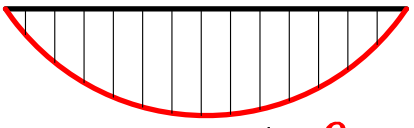
$$\frac{S}{b} = \frac{R^2}{3}$$

$$q_{max.} = \frac{Q}{\frac{\pi R^4}{4}} x \frac{R^2}{3}$$

$$= \frac{4Q}{3\pi R^2} = \frac{4}{3}\frac{Q}{A}$$

$$\boxed{q_{max.} = \frac{4}{3}\frac{Q}{A}}$$





$$q_{max.} = \frac{4}{3}\frac{Q_x}{A}$$

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لحساب الـ *Shear Stress* عند نقطة معينة :

$$q = \frac{Q}{I} \; x \; \frac{S}{b}$$

١- نأخذ خط عند هذه النقطة عموديا على الـ *Force* هذا الخط يقسم الشكل الى مساحتين .

٢- نحسب (S) لاى مساحة منهما .

٣- نحسب الـ I *(Moment of inertia)* للشكل كله حول المحور العمودى على الـ *Force* .

٤- نحسب b و هو العرض المصمت من الـ *Section* عند النقطة التى نحسب عندها الـ *Shear Stress* عمودى على الـ *Shear Force* .

٥- نعوض فى القانون العام $q = \frac{Q}{I} \; x \; \frac{S}{b}$

فى حالة القطاعات التى يتغير فيها العرض عند نقط محددة

نأخذ خط موازى للـ *Force* و نحسب الـ *Shear Stress* على هذا الخط عند النقط التالية

١ - أول و أخر نقطة فى القطاع .

٢ - نقط تغير العرض و نحسب قبل التغير و بعده .

٣ - نقطة الـ *C.g* .

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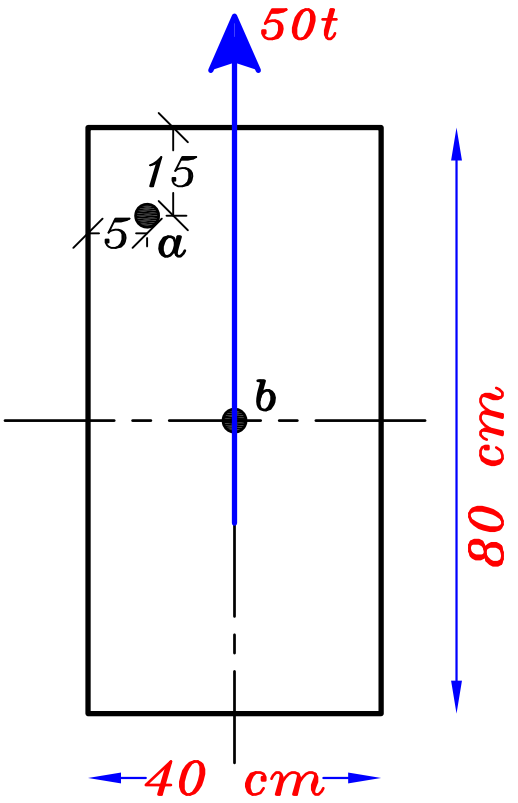
Example:

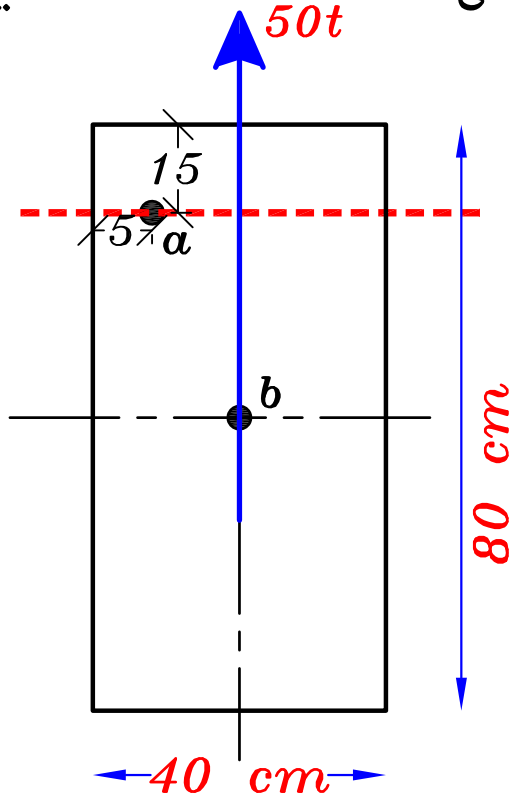
For the shown section find the shear stresses at points (a&b).



For Point (a)

هذا الـ Section له عرض ثابت

١- نأخذ خط عند هذه النقطة عموديا على الـ Force هذا الخط يقسم الشكل الى مساحتين .



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٢- نحسب (S) لاى مساحة منهما .

$$S = (15 \ x \ 40) \ x \ (32.5) = 19500 \ cm^3$$

٣- نحسب الـ $I \ (Moment \ of \ inertia)$ للشكل كله حول المحور العمودى على الـ $Force$.

$$I_x = (40) \ x \ (80)^3 \ / \ 12 \ = 1706666.67 \ cm^4$$

٤- نحسب b و هو العرض المصمت من الـ $Section$ عند النقطة التى نحسب عندها الـ $Shear \ Stress$ عمودى على الـ $Shear \ Force$.

$$b = 40 \ cm$$

٥- نعوض فى القانون العام $q = \frac{Q}{I} \ x \ \frac{S}{b}$

$$q_a = \frac{50}{1706666.67} \ x \ \frac{19500}{40} = \mathbf{0.01428} \ t/Cm^2$$

لا نحتاج لعمل هذه الخطوات عند نقطة b لان $C.g$ المستطيل محفوظ عنده قيمة الـ $Shear \ Stress$ $q = 1.5Q/A$

و قد تم اثبات ذلك سابقا و ممكن استخدام هذه القيمة مباشرة

For Point (b)

$$q_{max.} = 1.5 \ \frac{Q}{A} = 1.5 \ \frac{50}{40x80} = \mathbf{0.02343} \ t/Cm^2$$

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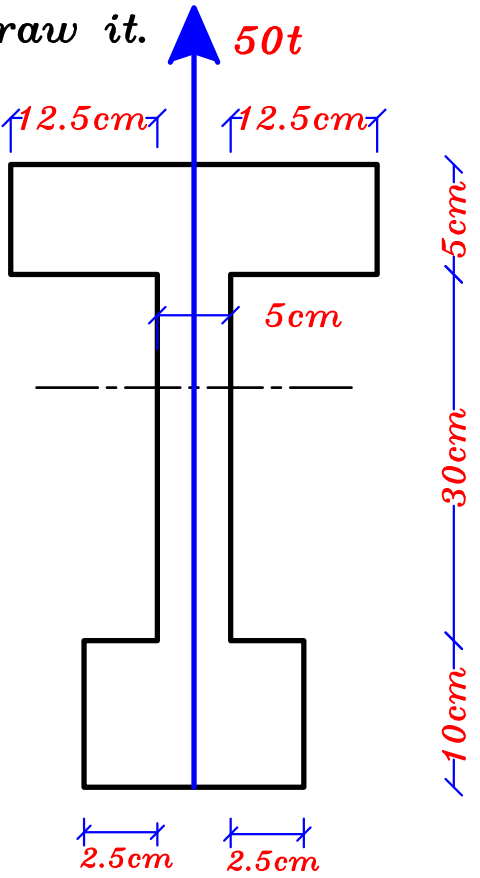
Example:

For the shown section find the max. shear stresses and its position and then draw it.



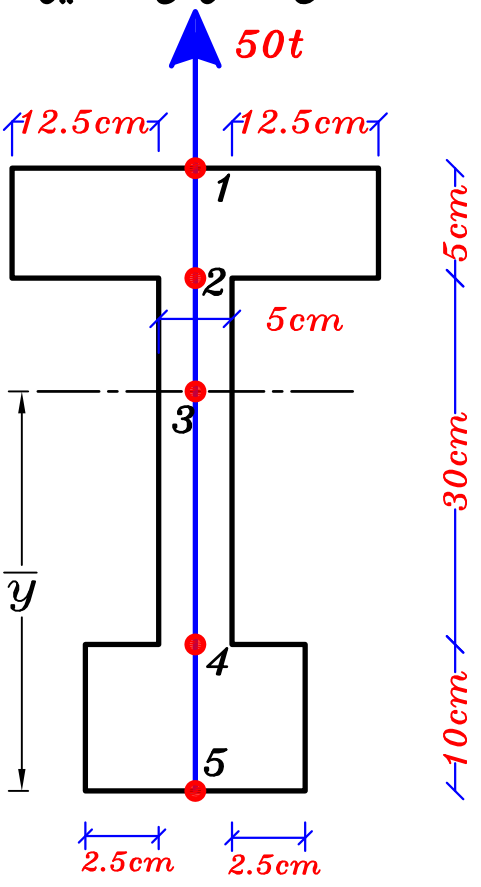
في هذا المثال العرض متغير عند نقط معينة و لذلك لابد أولا من تحديد الـ C.g

$$\Sigma A = 100 + 150 + 150 = 400 \ cm^2$$

$$\overline{Y} = \frac{\Sigma A \ x \ Y}{\Sigma A}$$

$$\overline{Y} = \frac{(100)(5)+(150)(25)+(150)(42.5)}{400}$$

$$= 26.5625 \ cm$$



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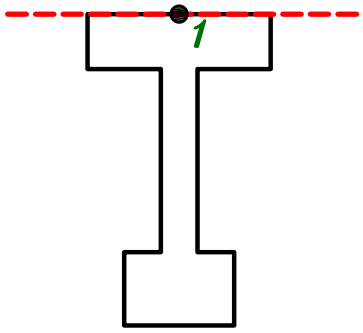
Point (1)

١- نأخذ خط عند هذه النقطة عموديا على الـ *Force* هذا الخط يقسم الشكل الى مساحتين .

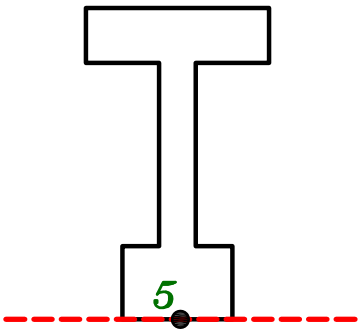


$$S = 0$$

$$q_1 = \frac{Q}{I} x \frac{S}{b} = \frac{Q}{I} x \frac{0}{b} = 0$$

Point (5)

و بالمثل نقطة 5



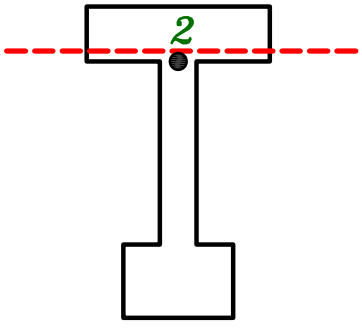
$$q\ @\ 1 = q\ @\ 5 = 0$$

ملحوظة هامة جدا

لو العرض متغير نقوم بحساب الـ *Stresses* عند النقطة التى يتغير عندها العرض مرتين مرة قبل التغير و مرة بعده .

Point (2) Up

١- نأخذ خط عند هذه النقطة عموديا على الـ *Force* هذا الخط يقسم الشكل الى مساحتين .



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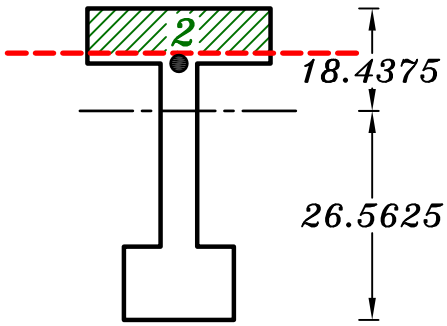
٢- نحسب (S) لاى مساحة منهما .



$$S = 30 \times 5 \times 15.94 = 2391 \ cm^3$$

٣- نحسب الـ I $(Moment\ of\ inertia)$ للشكل كله حول المحور العمودى على الـ Force .

$$I_x = [\ (10)\ (10)^3/12 + (100)\ (26.5625-5)^2]$$

$$+ [\ (5)\ (30)^3/\ 12 + (150)\ (26.5625-25)^2\ ]$$

$$+ [\ (30)\ (5)^3/\ 12 + (150)\ (26.5625-42.5)^2\ ]$$

$$= 97356.773 \ cm^4$$

٤- نحسب b و هو العرض المصمت من الـ Section عند النقطة التى نحسب عندها الـ Shear Stress عمودى على الـ Shear Force .

$$b = 30 \ cm$$

٥- نعوض فى القانون العام $q = \frac{Q}{I} \times \frac{S}{b}$

$$q_{\substack{2 \\ Up}} = \frac{50}{97356.773} \times \frac{2391}{30} = 0.04093 \ t/Cm^2$$

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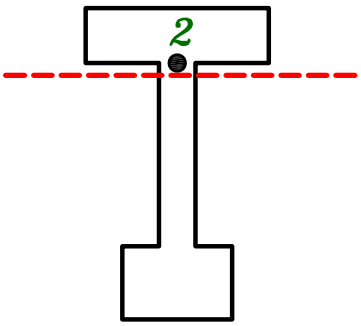
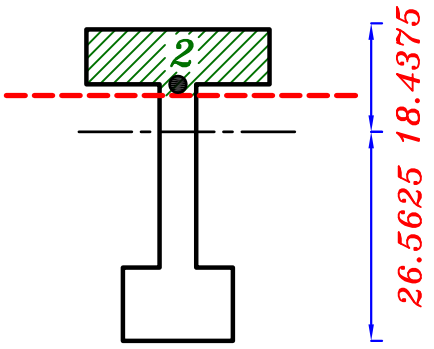
Point (2) down

١- نأخذ خط عند هذه النقطة عموديا على الـ Force هذا الخط يقسم الشكل الى مساحتين .



٢- نحسب (S) لاى مساحة منهما .



$$S = 30 \times 5 \times 15.94 = 2391 \ cm^3$$

نلاحظ أن الـ S لها نفس القيمة و لذلك فان الـ S لا تتغير عند نفس النقطة .

٣- نحسب الـ I (Moment of inertia) للشكل كله حول المحور العمودى على الـ Force و هو محسوب فى الخطوة السابقة .

٤- نحسب b و هو العرض المصمت من الـ Section عند النقطة التى نحسب عندها الـ Shear Stress عمودى على الـ Shear Force .

$$b = 5 \ cm$$

٥- نعوض فى القانون العام $q = \frac{Q}{I} \times \frac{S}{b}$

$$q_{\substack{2 \\ down}} = \frac{50}{97356.773} \times \frac{2391}{5} = 0.2456 \ t/Cm^2$$

لاحظ ان الفرق الوحيد بين النقطتين 2up&2down هوعرض الـ Sec.

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Point (3)

$$q = \frac{Q}{I} x \frac{S}{b}$$

$$q_3 = \frac{50}{97356.773} x \frac{[\,(30x5x15.94) + (13.44x5x6.72)\,]}{5}$$

$$= 0.2719\ t/Cm^2$$

Point (4)

$$q_{4\ Up} = \frac{50}{97356.773} x \frac{(10x10x21.56)}{5}$$

$$= 0.2211\ t/Cm^2$$

$$q_{4\ down} = \frac{50}{97356.773} x \frac{(10x10x21.56)}{10}$$

$$= 0.110\ t/Cm^2$$

ملحوظة هامة

فى هذا النوع من المسائل اذا كان اقل عرض عند الـ $C.g$ يكون $q_{max.}$ عند الـ $C.g$ و اذا لم يكن عند الـ $C.g$ تكون الـ $q_{max.}$ عند أقرب أقل عرض من الـ $C.g$.

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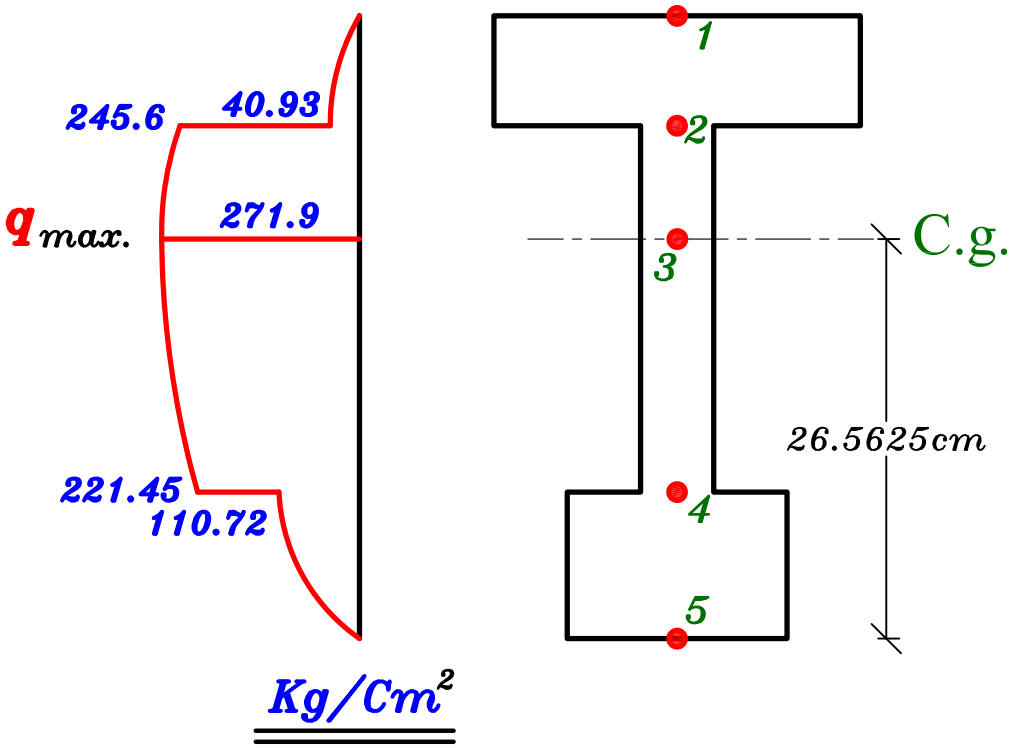

Shear stress distribution

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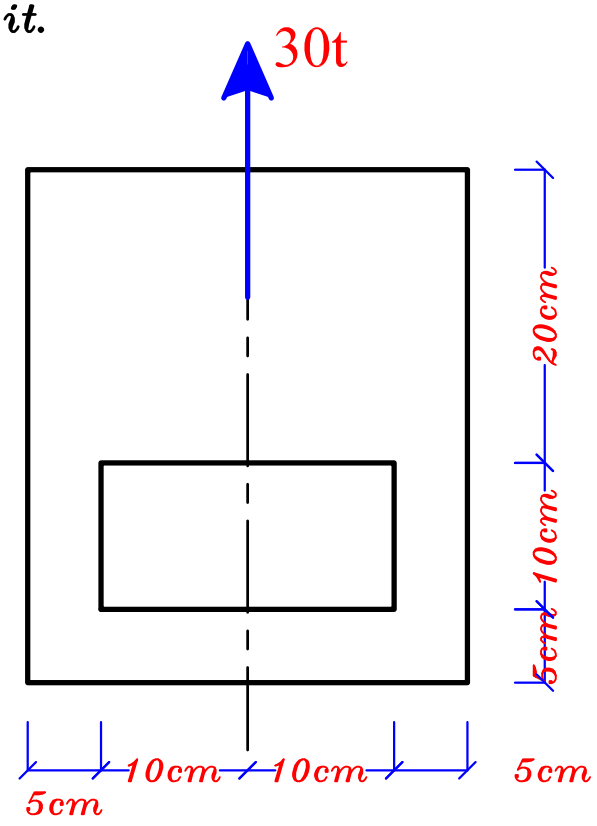
Example:

For the shown section find the max. shear stresses and its position and then draw it.



فى هذا المثال العرض متغير عند نقط معينة و لذلك لابد أولا من تحديد الـ C.g

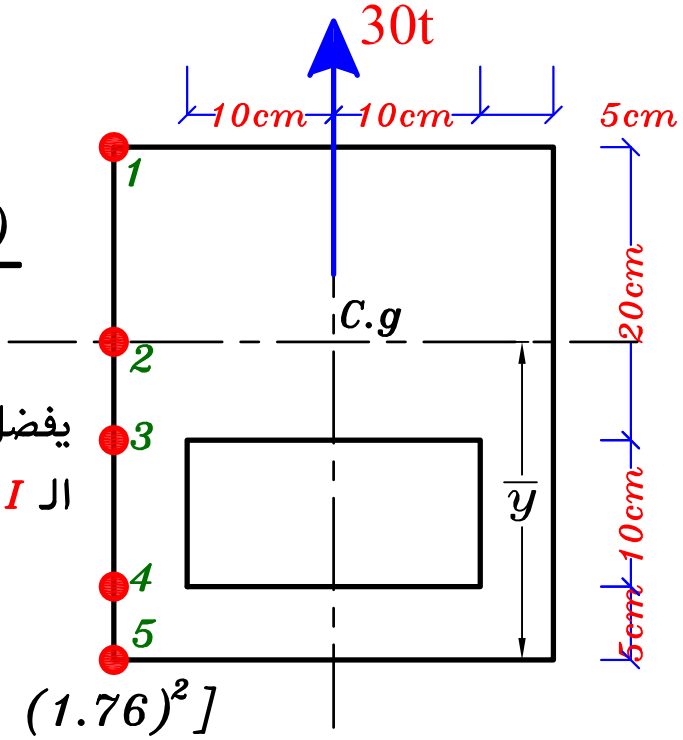
$$\Sigma A = 30x35 - 20x10 = 850\ cm^2$$

$$\bar{Y} = \frac{\Sigma A\ x\ Y}{\Sigma A}$$

$$\bar{Y} = \frac{(30)(35)(17.5) - (20)(10)(10)}{850}$$

$$= 19.26\ cm$$

يفضل حساب خطوة رقم ٣ فى البداية لان الـ I ثابتة عند كل النقاط فى أى مسألة .



$$I_x = [\ (35)\ (30)^3/12 + (1050)\ (1.76)^2\ ]$$

$$- [\ (20)\ (10)^3/\ 12 + (200)\ (9.26)^2\ ]$$

$$= 91623.8\ cm^4$$

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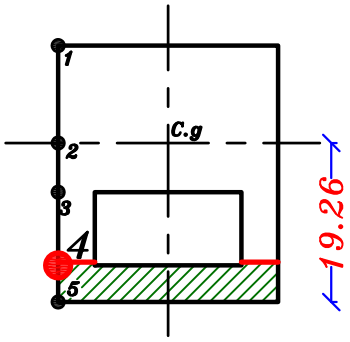
Point (4)

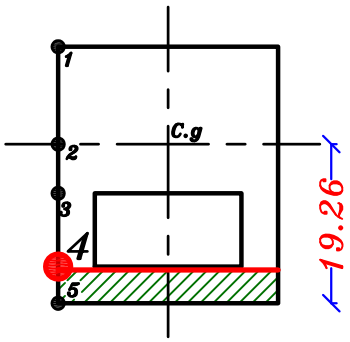
$$q_{4\ Up} = \frac{30}{96123.8} x \frac{[\ (30x5x16.76)\ ]}{10}$$

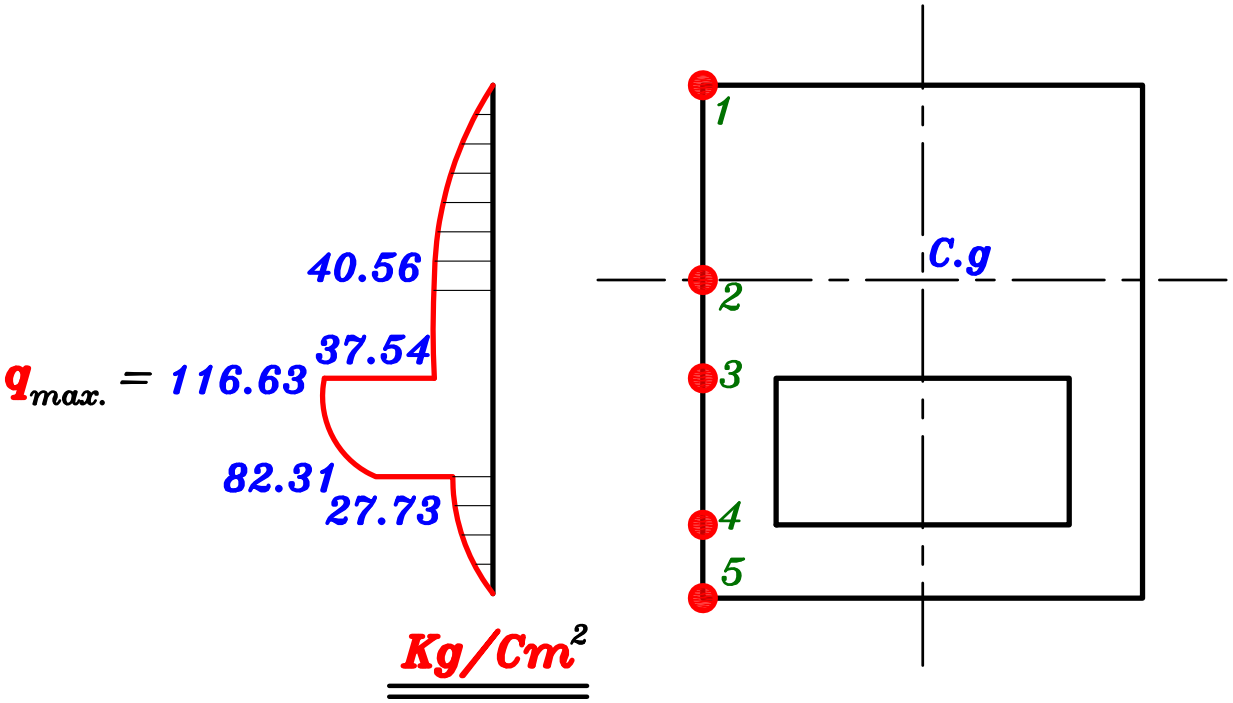
$$= 0.08231\ t/Cm^2$$



$$q_{4\ down} = \frac{30}{96123.8} x \frac{[\ (30x5x16.76)\ ]}{30}$$

$$= 0.02773\ t/Cm^2$$





Shear stress distribution

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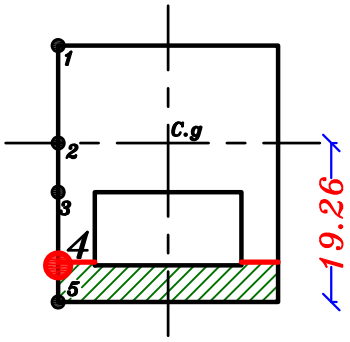
Point (4)

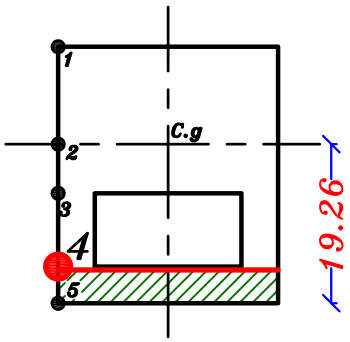
$$q_{4\ Up} = \frac{30}{96123.8} x \frac{[\ (30x5x16.76)\ ]}{10}$$

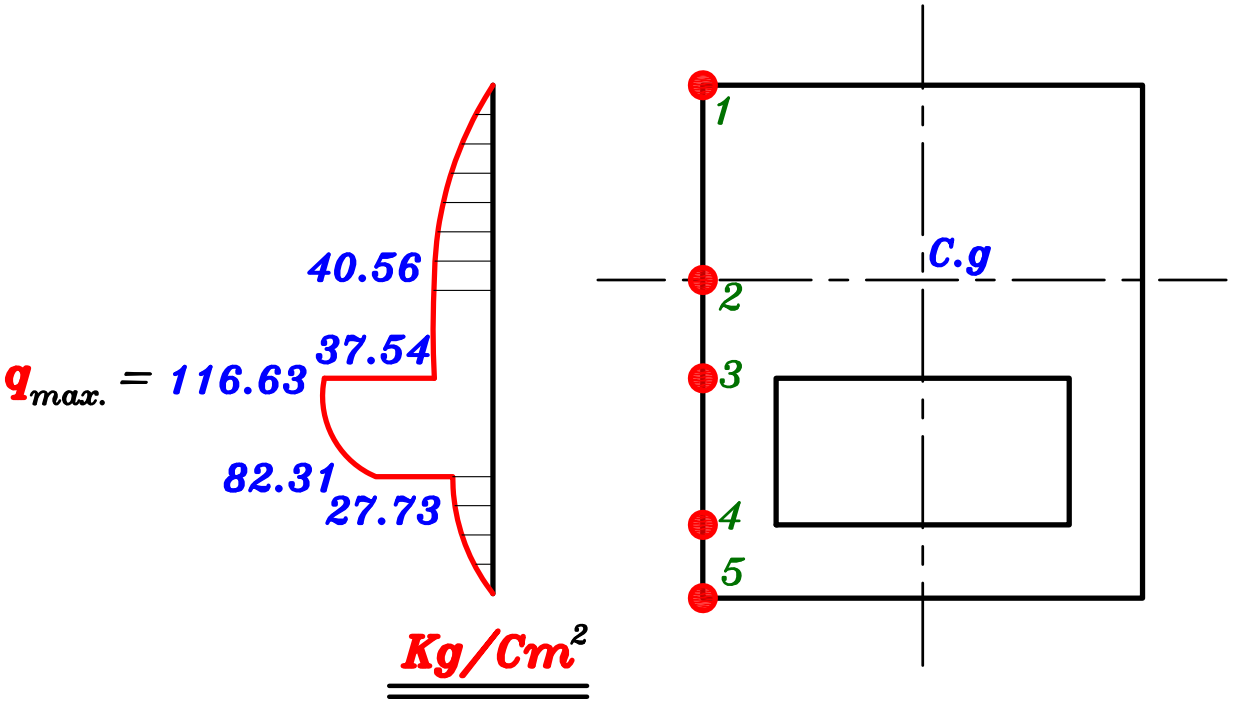
$$= 0.08231\ t/Cm^2$$



$$q_{4\ down} = \frac{30}{96123.8} x \frac{[\ (30x5x16.76)\ ]}{30}$$

$$= 0.02773\ t/Cm^2$$





Shear stress distribution

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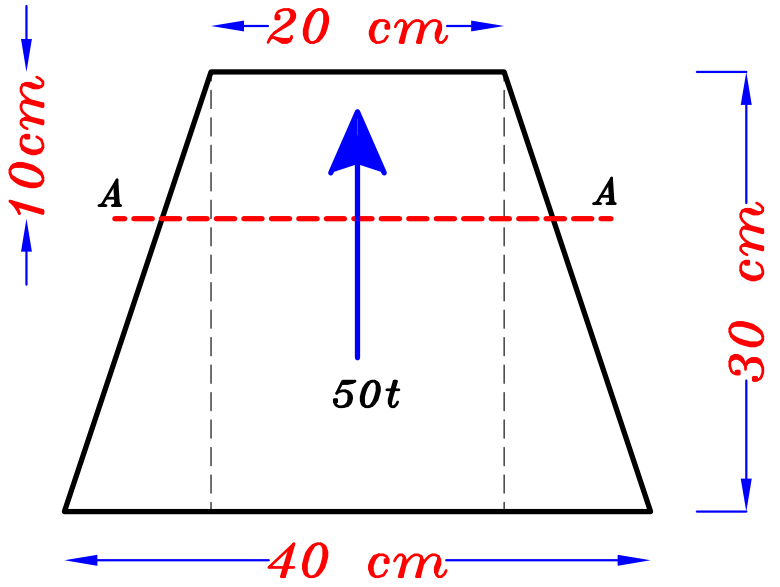
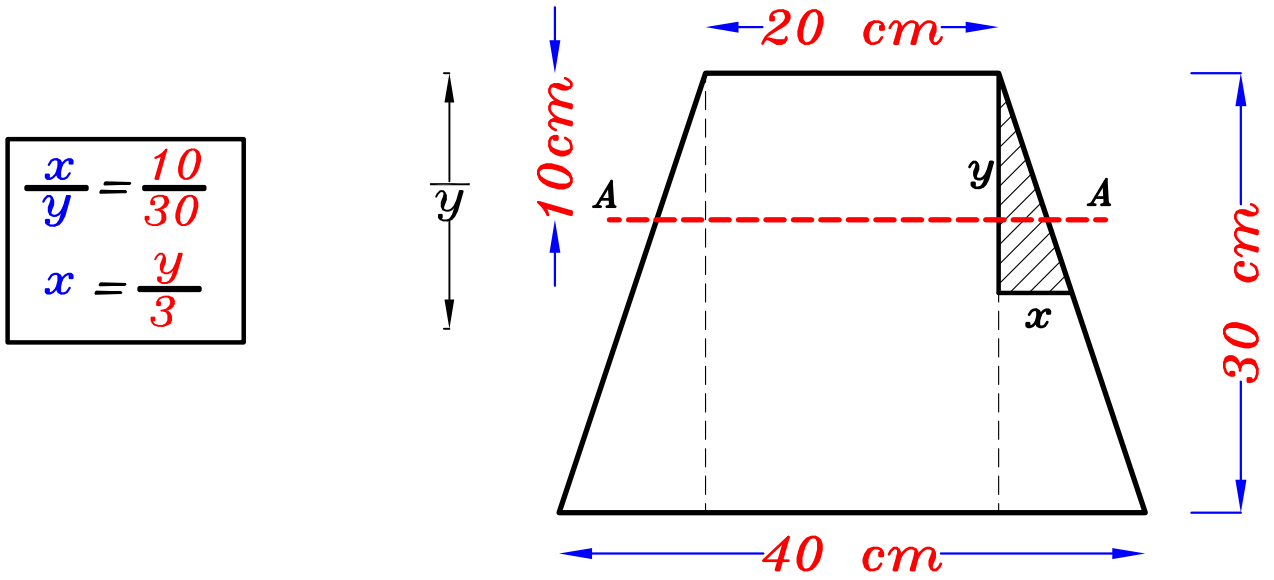
Example:

For the shown section find the shear stresses at the section (A–A).





$\Sigma A = (40+20)/2 \times 30 = \mathbf{900}\ cm^2$

$$\bar{Y} = \frac{\Sigma A \ x \ Y}{\Sigma A}$$

$$\bar{Y} = \frac{(20)(30)(15)+(2)(1/2)(10)(30)(10)}{900}$$

$= \mathbf{16.67\ cm}$

$S\ @A–A = [(20x10x(16.67–5)]$

$+[2x(0.5x10x10/3)x(16.67–6.67)]$

$\mathbf{=2667.33\ cm^3}$

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$$I_x = [\ (20)\ (30)^3/12 + (20x30)\ (1.67)^2\ ]$$

$$+\ 2\ [\ (10)\ (30)^3/\ 36 + (0.5x10x30)\ (3.33)^2\ ]$$

$$= 65000\ cm^4$$

$$b\ @A-A = 20 + 2\ x\ \frac{1}{3}\ x\ 10 = 26.67\ cm$$

$$q = \frac{Q}{I}\ x\ \frac{S}{b}$$

$$q_{2\ down} = \frac{50}{65000}\ x\ \frac{2667.33}{26.67} = 0.0769\ t/Cm^2$$

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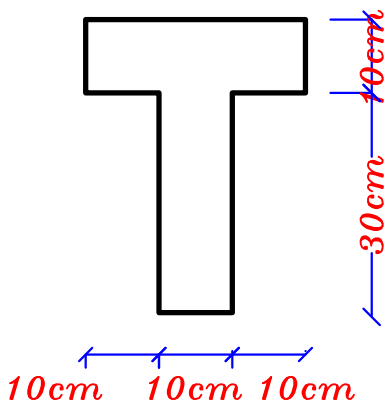
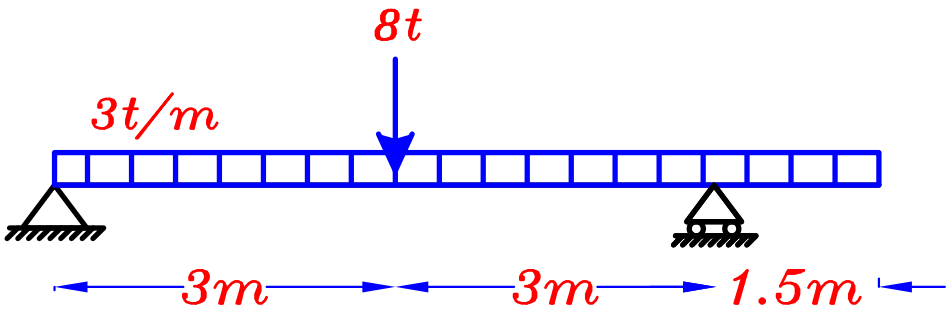
Example:

For the shown beam draw the shear stress distribution.



Beam Section

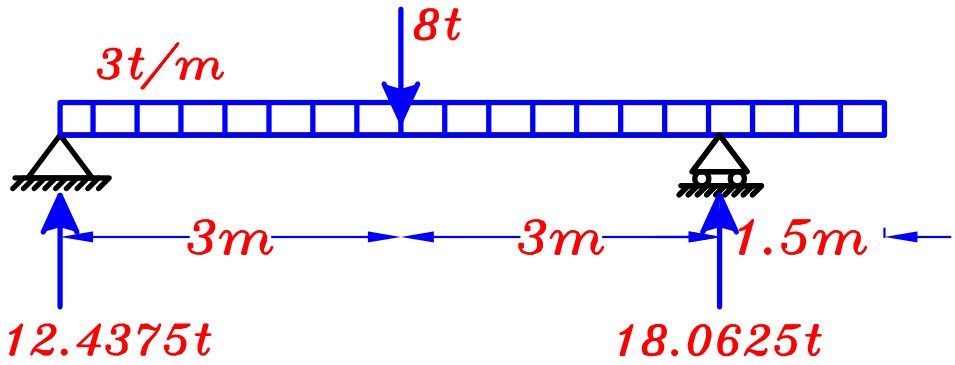


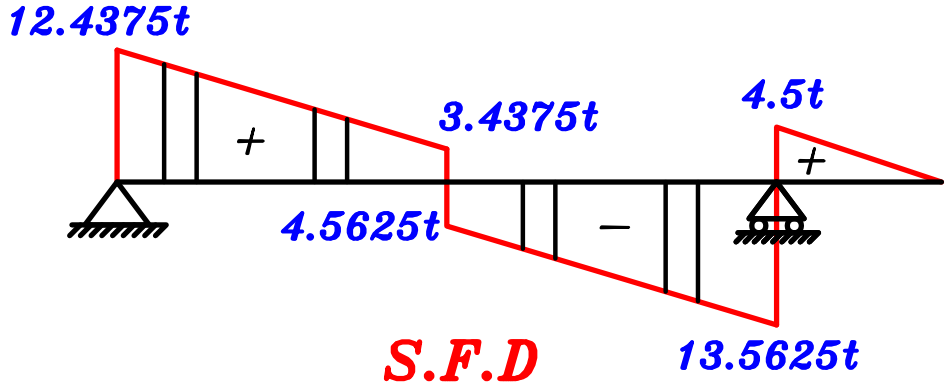
من معادلة الShear stress فنجد أنه كلما زادت قيمة الShear force تزداد قيمة الShear stress .

$$q = \frac{Q}{I} x \frac{S}{b}$$

و لذلك فان لم يكن محددا الSection الذى من المفروض حساب و رسم توزيع الShear stress فنحسب عند أكبر Section به Shear force حيث أنه سيكون أكبر Section عليه Shear stress .

و لمعرفة أكبر Section عليه Shear force سوف نرسم الS.F.D.





S.F.D

$Q_{max.} = 13.5625t$

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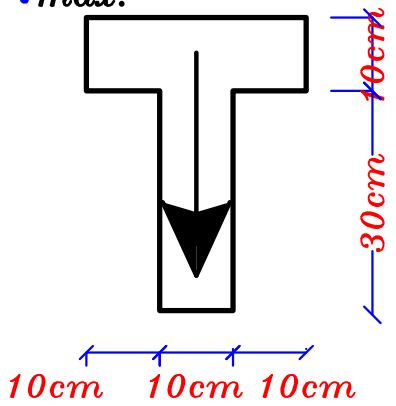
Beam Section

$Q_{max.} = 13.5625t$



$$\Sigma A = 10x30 + 10x30 = 600\ cm^2$$

$$\overline{Y} = \frac{\Sigma A\ x\ Y}{\Sigma A}$$

$$\overline{Y} = \frac{(10)(30)(15)+(10)(30)(35)}{600}$$

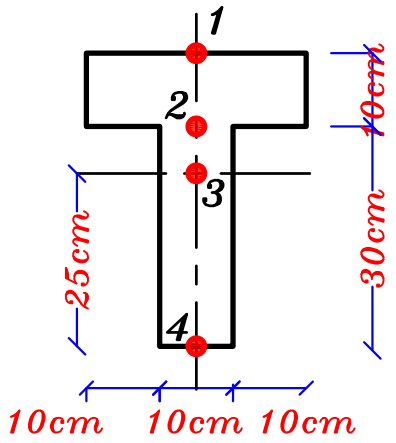
$$= 25\ cm$$

$$I_x = [\ (30)\ (10)^3/12\ +\ (30x10)\ (10)^2\ ]$$

$$+\ [\ (10)\ (30)\ /^3\ 12\ +\ (30)\ (10)\ (10)^2\ ]$$

$$= 85000\ cm^4$$

Point (1) & (4)



$S = 0$

$$q_1 = q_4 = \frac{Q}{I} x \frac{S}{b} = \frac{Q}{I} x \frac{0}{b} = 0$$

Point (2)

$$q = \frac{Q}{I} x \frac{S}{b}$$

$$q_{2\ Up} = \frac{13.5625}{85000} x \frac{(30x10x10)}{30}$$

$$= 0.016\ t/Cm^2$$

$$q_{2\ down} = \frac{13.5625}{85000} x \frac{(30x10x10)}{10}$$

$$= 0.0478\ t/Cm^2$$

Point (3)

$$q_3 = \frac{13.5625}{85000} x \frac{(25x10x12.5)}{10}$$

$$= 0.0498\ t/Cm^2$$

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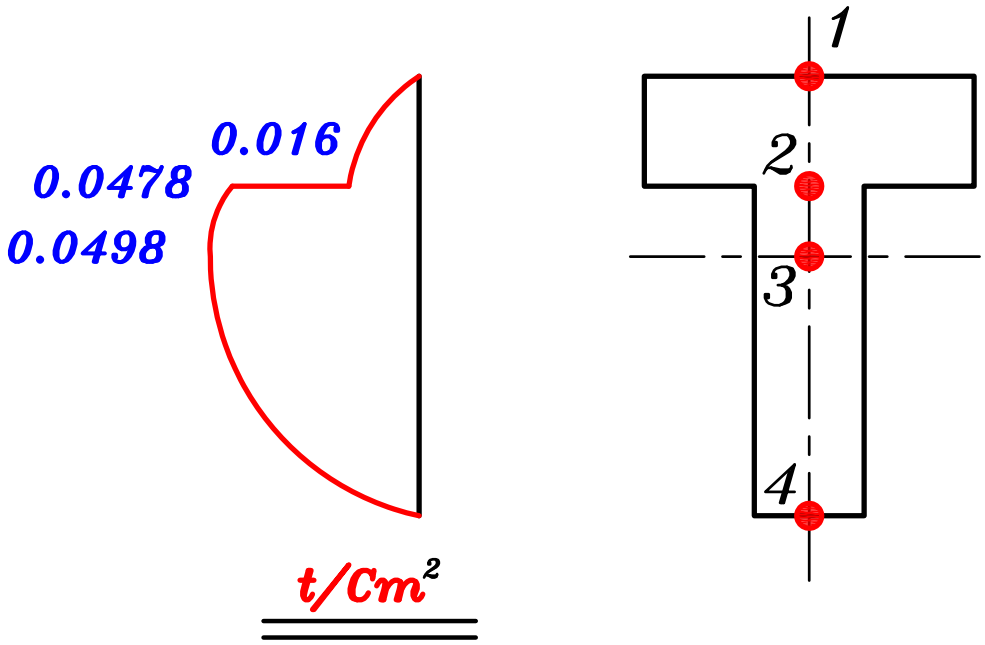
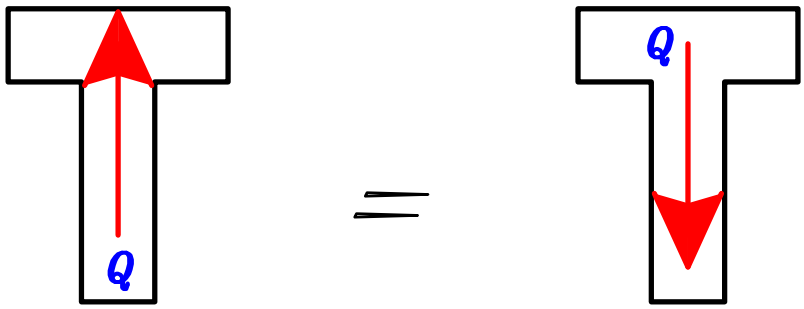


Shear stress distribution

ملحوظة هامة

اتجاه الـ *Shear force* سواء لاعلى أو أسفل لا يؤثر على أى شئ فى الحسابات



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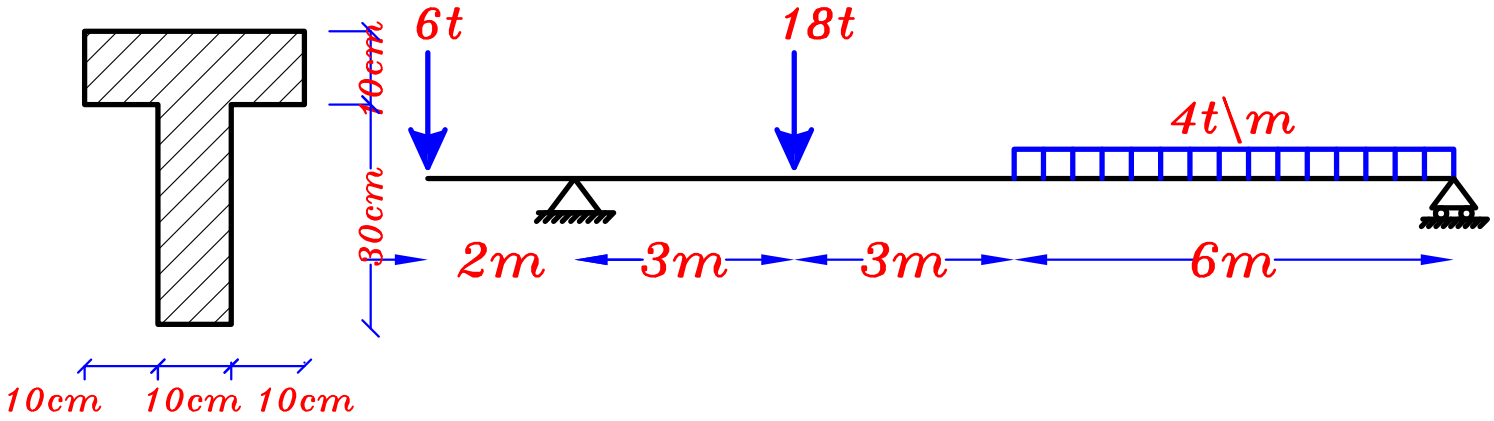
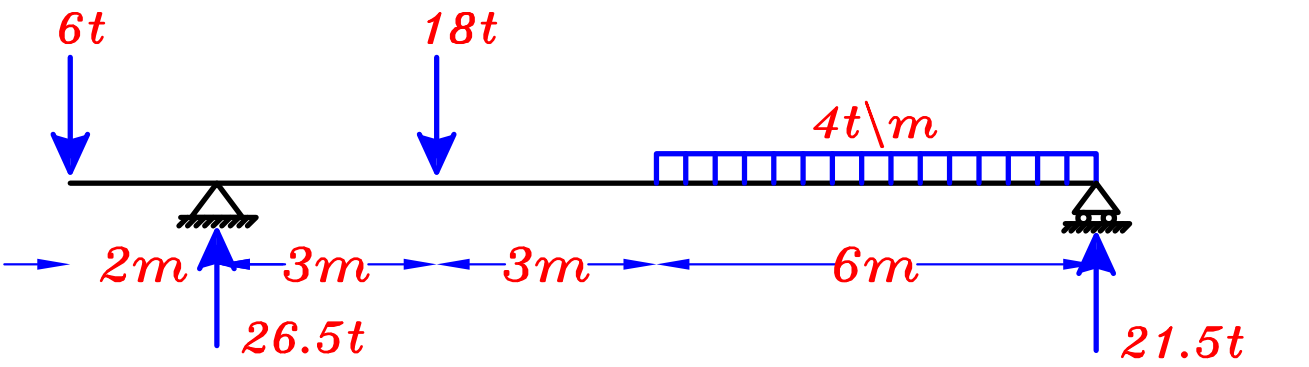
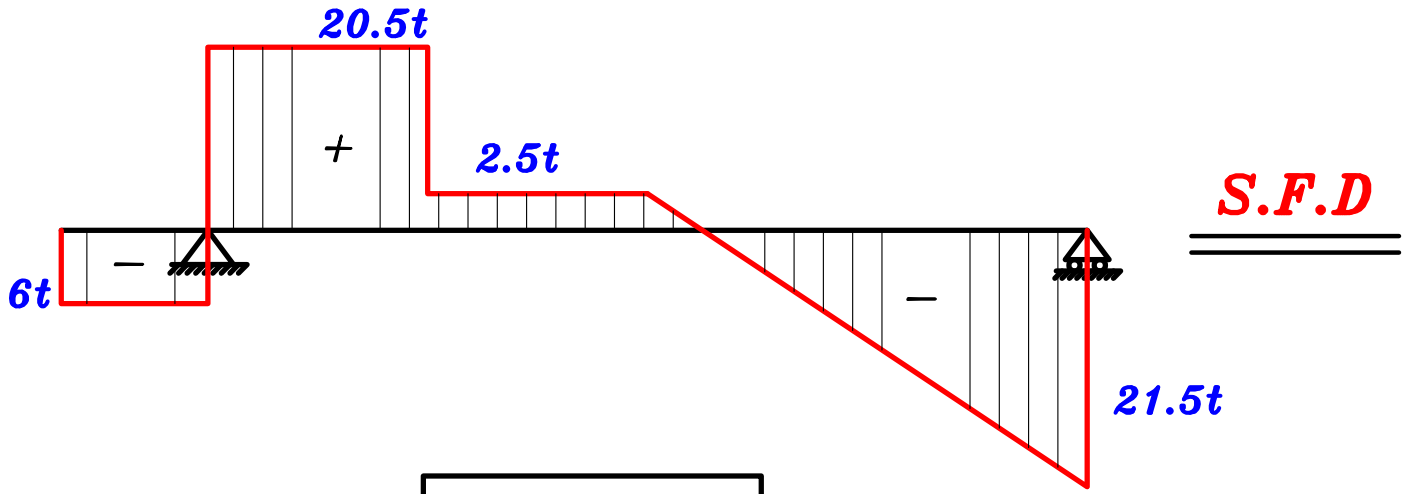
Example:

For the shown beam draw the S.F.D then find the max. shear stresses and its position at the beam section and then draw it.



Beam Section





S.F.D

$Q_{max.} = 21.5t$

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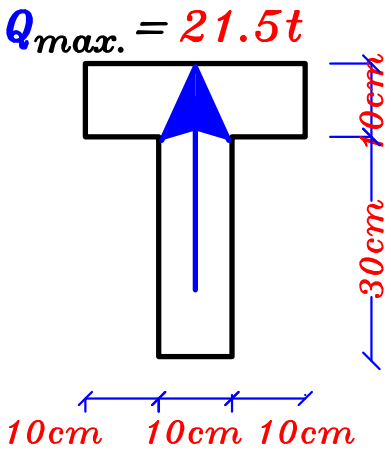
Beam Section



$$\Sigma A = 10x30 + 10x30 = 600\ cm^2$$

$$\overline{Y} = \frac{\Sigma A\ x\ Y}{\Sigma A}$$

$$\overline{Y} = \frac{(10)(30)(15)+(10)(30)(35)}{600}$$

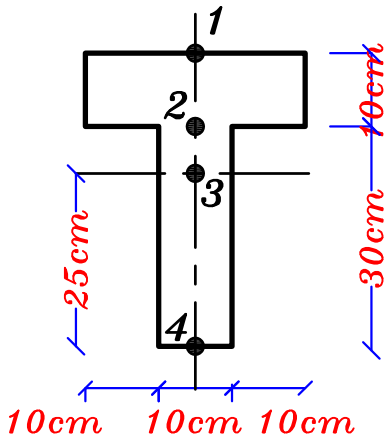
$$= 25\ cm$$

$$I_x = [\ (30)\ (10)^3/12\ +\ (30x10)\ (10)^2\ ]$$

$$+\ [\ (10)\ (30)\ /^3\ 12\ +\ (30)\ (10)\ (10)^2\ ]$$

$$= 85000\ cm^4$$

Point (1) & (4)



$$S = 0$$

$$q_1 = q_4 = \frac{Q}{I} x \frac{S}{b} = \frac{Q}{I} x \frac{0}{b} = 0$$

Point (2)

$$q = \frac{Q}{I} x \frac{S}{b}$$

$$q_{2\ Up} = \frac{21.5}{85000} x \frac{(30x10x10)}{30}$$

$$= 0.02593\ t/Cm^2$$

$$q_{2\ down} = \frac{21.5}{85000} x \frac{(30x10x10)}{10}$$

$$= 0.07588\ t/Cm^2$$

Point (3)

$$q_3 = \frac{21.5}{85000} x \frac{(25x10x12.5)}{10}$$

$$= 0.07194\ t/Cm^2$$

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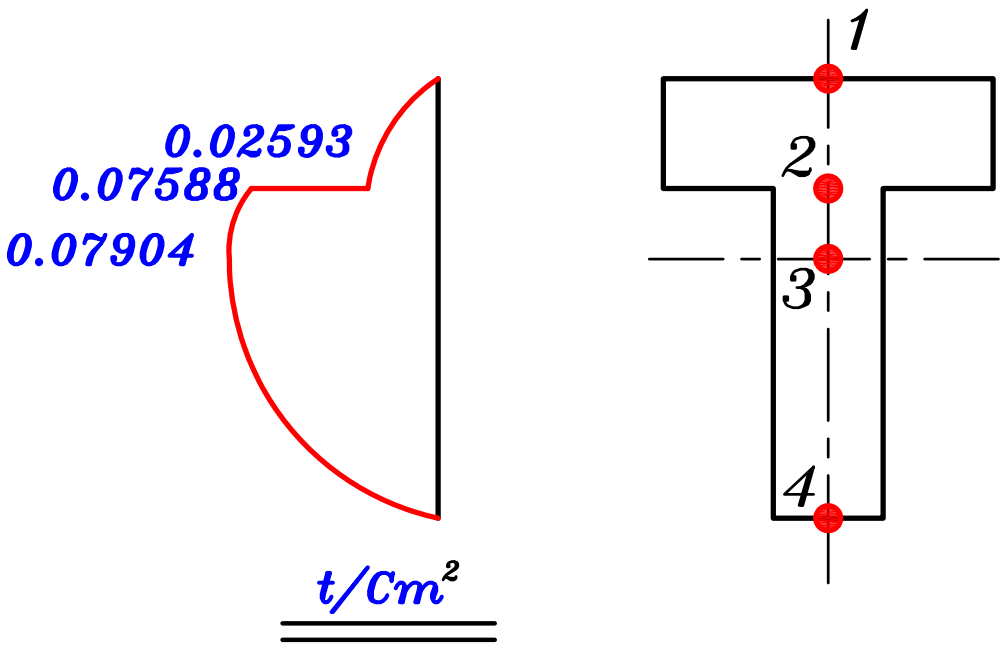

Shear stress distribution

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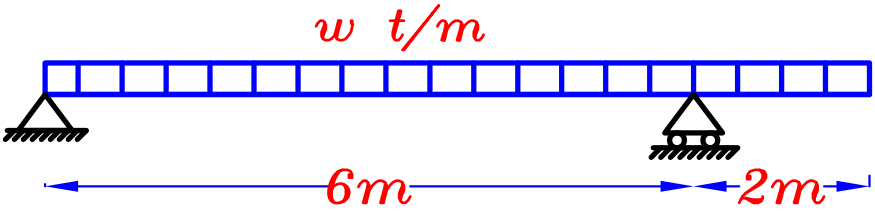
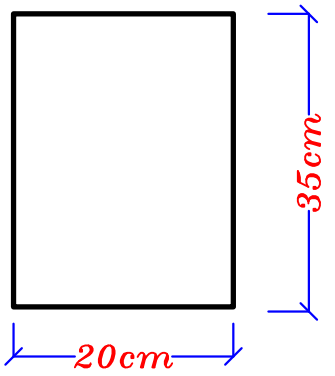
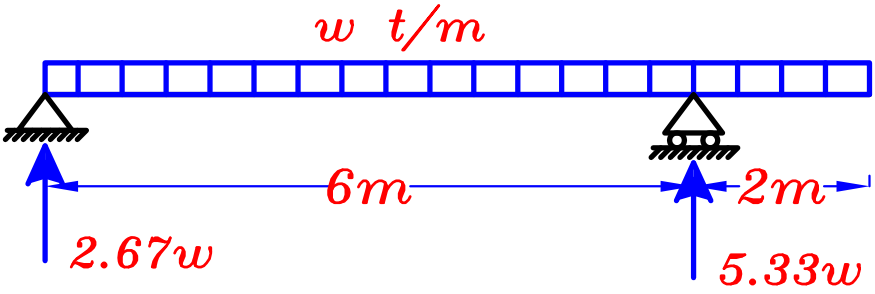
Example:

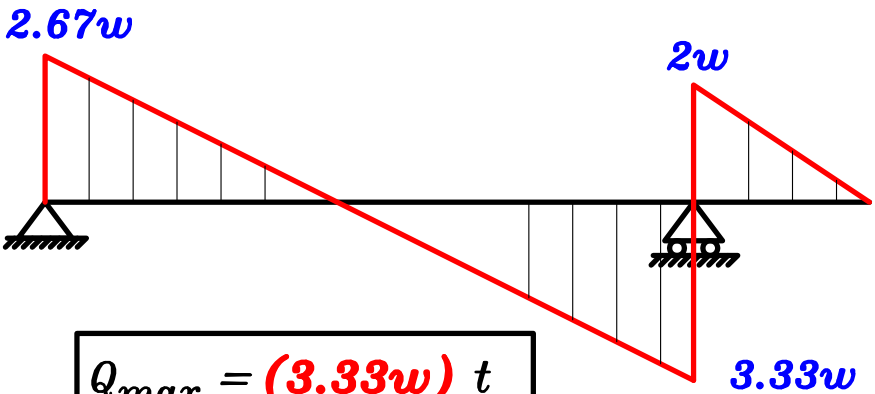
For the shown beam find the maximum value for the load (w) so that the maximum shear stress dosen't exceed 0.9 t/cm².





Beam Section





S.F.D

$Q_{max.} = (3.33w)\ t$

For Rectangular section: $q_{max.}$ *is at C.g*

$$q_{max.} = 1.5\ \frac{Q}{A}$$

$$0.90\ t/Cm^2 = 1.5\ \frac{3.33w}{20x35}$$

$$w = 126\ t\backslash m$$

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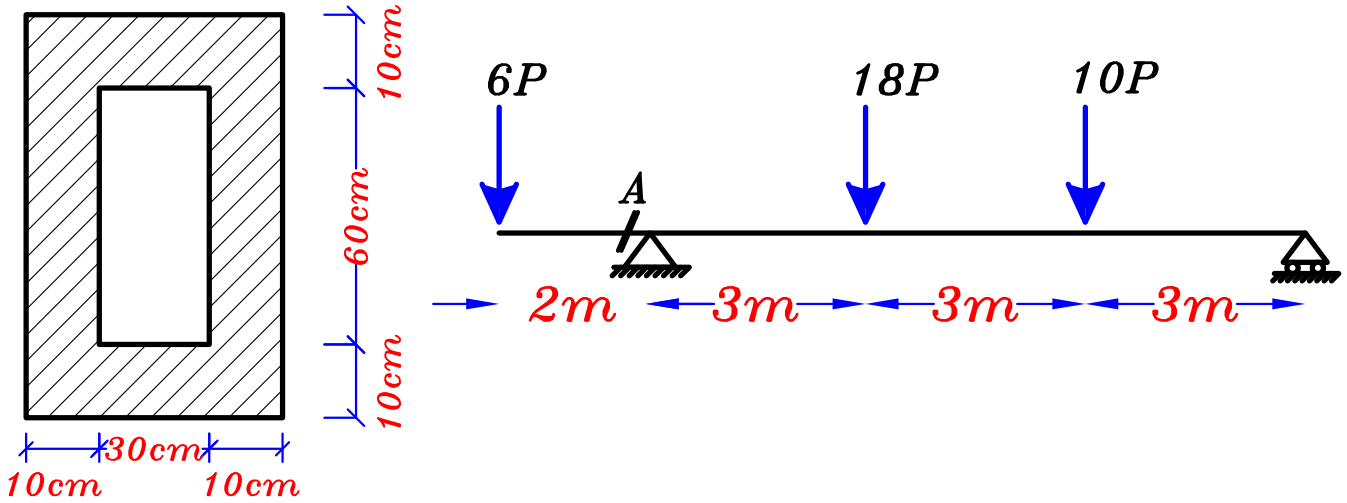
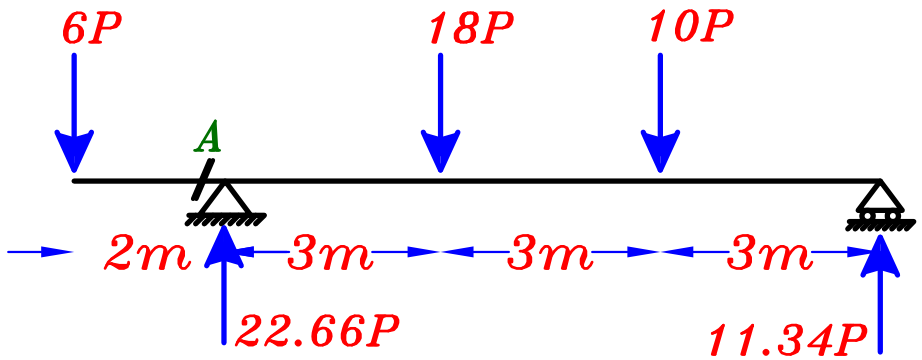
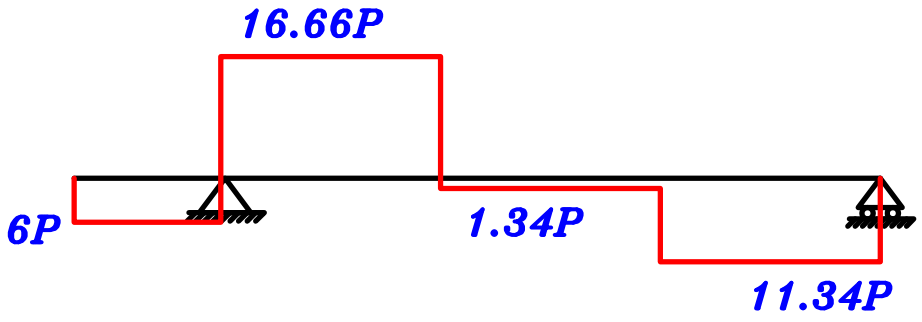
Example:

For the shown beam find the maximum value for the load (P) so that the maximum shear stress dosen't exceed 0.8 t/cm². then find the maximum shear stress at point (A).





S.F.D



$Q_{A-A} = (6P)\ t$

$Q_{max.} = (16.66P)\ t$

أقل عرض فى الـ Section عند الـ C.g و بالتالى يكون أكبر Shear stress عند الـ C.g .

$I_x = [\ (50)\ (80)^3/12\ ] - [\ (30)\ (60)^3/\ 12\ ]$

$= 1593333\ cm^4$

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$$q = \frac{Q}{I} x \frac{S}{b}$$

$$q_{max} = \frac{16.66P}{1593333} x \frac{(50x10x35)+(30x10x15x2)}{20}$$

$$= \mathbf{0.01386P}\ t/Cm^2$$

$$\mathbf{0.01386P}\ t/Cm^2 = 0.80$$

$$\boxed{\mathbf{P} = \mathbf{57.72}\ t}$$

At Sec. A–A

$$Q_{A-A} = (6P)\ t = (6x57.72) = \mathbf{346.32t}$$

$$q = \frac{Q}{I} x \frac{S}{b}$$

$$q_{A-A} = \frac{346.32}{1593333} x \frac{(50x10x35)+(30x10x15x2)}{20}$$

$$= \mathbf{0.288}\ t/Cm^2$$

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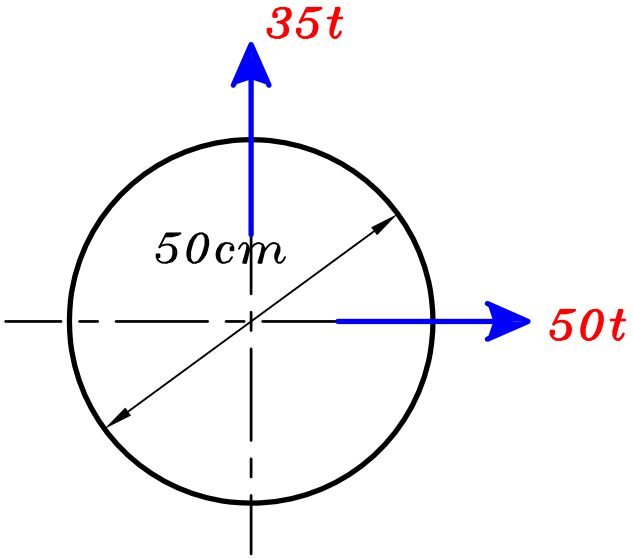
Example:

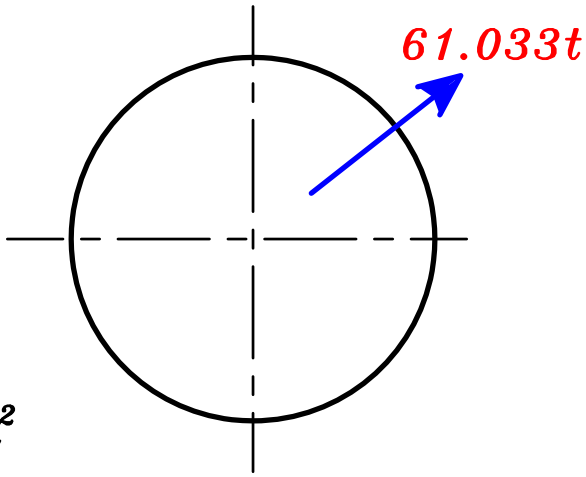
For the shown section find the max. shear stresses and its position and then draw it.



من الممكن حساب المحصلة و الشغل عليها

$$Q_R = \sqrt{Q_x^2 + Q_y^2}$$

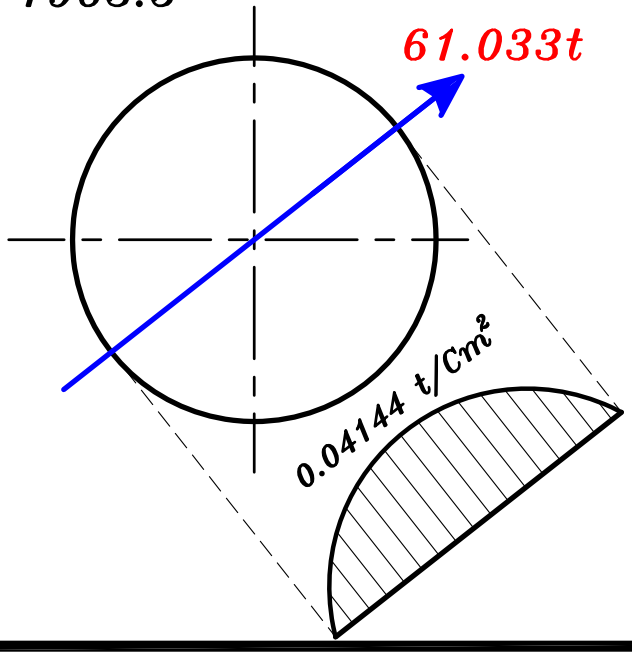
$$Q_R = \sqrt{50^2 + 35^2} = 61.033t$$



$$A = \pi R^2 = \pi 25^2 = 1963.5\ Cm^2$$

$$q_{max.} = \frac{4}{3}\frac{Q}{A}$$

$$= \frac{4}{3}\frac{61.033}{1963.5} = 0.04144\ t/Cm^2$$



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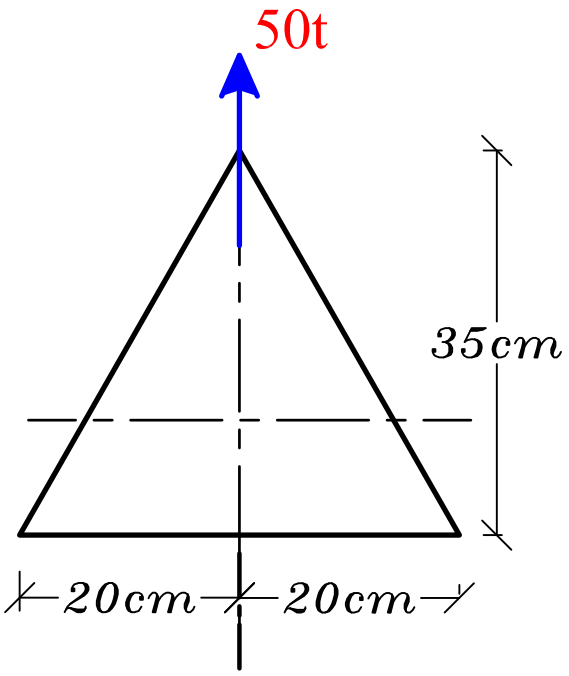
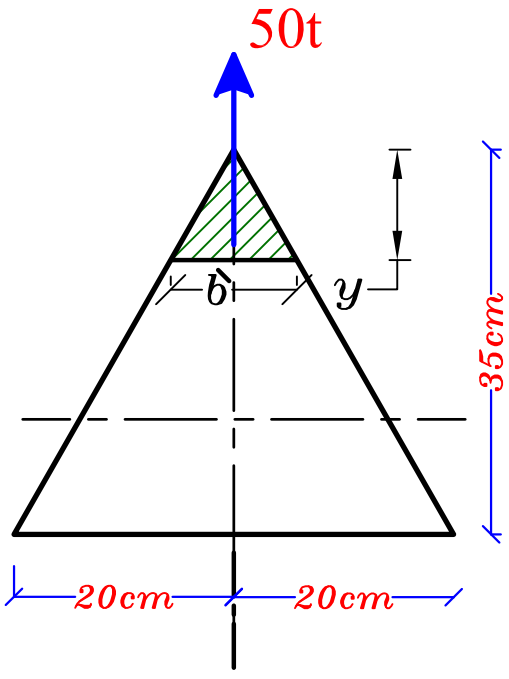
EXAMPLE

For the shown section find the max. shear stresses and its position and then draw it.



Q / I = constant



q α (S/b)

q max. @ (S/b) max.

S = [(1 / 2) (Y) (b\)] [23.33 - 2 / 3 y]

S = (b\) (11.66 y - 1 / 3 y^2)

S / b = 11.66 y - 1 / 3 y^2

d(S/b) / d(y) = 11.66 - 2 / 3 y

d(S/b) / d(y) = 0

y = 17.5 cm

(S / b) max. = 11.66 x 17.5 - 1 / 3 x 17.5^2 = 102.07166 cm^2

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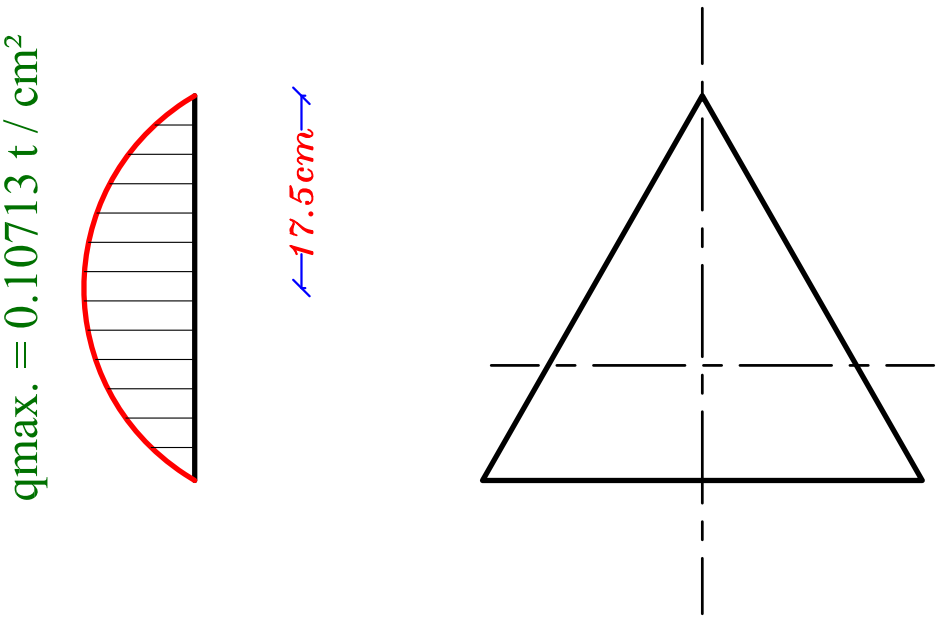
q max. = (Q / I) (S / b) max.

$Ix = bh^3 / 36 = (40) (35)^3 / 36 = 47638.88\ cm^4$

q max. = (50 / 47638.88) (102.07166) = 0.10713 t / cm²

q max. = 0.10713 t / cm²



Shear stress distribution (S . S . D)

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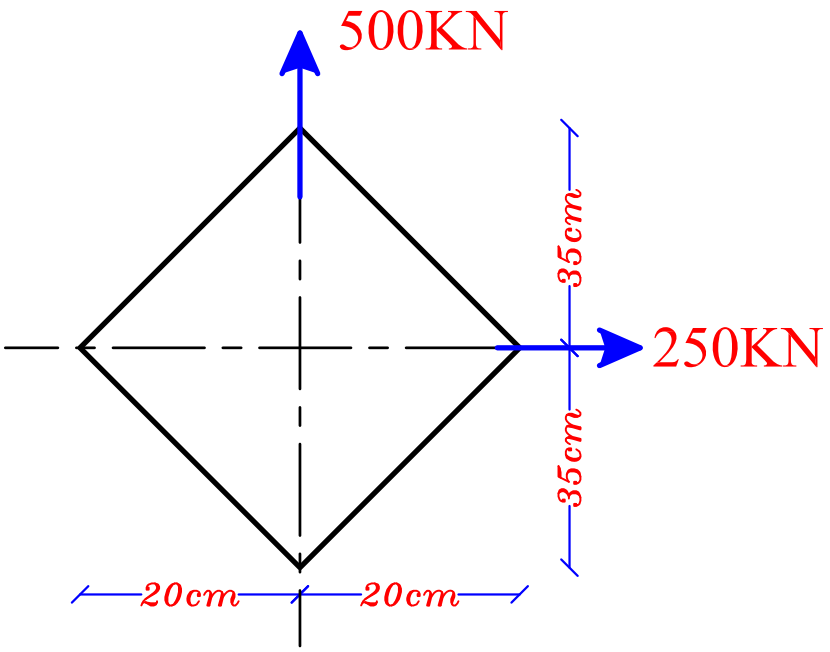
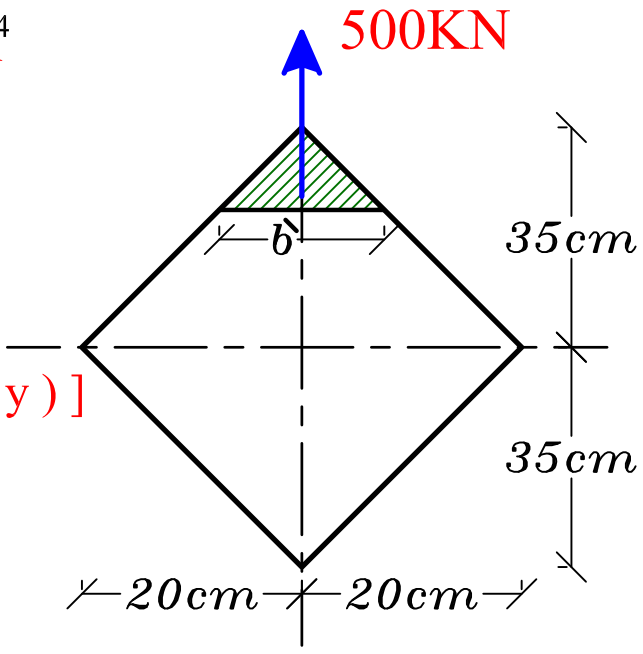
EXAMPLE

For the shown section find the max. shear stresses and its position and then draw it.



$I_x = (2)(20)(70)^3 / 48 = 285833\ cm^4$

$I_y = (2)(35)(40)^3 / 48 = 93333\ cm^4$



For Q_y

$S = [(1 / 2) (y) (b\)] [35 - (2 / 3 \ y)]$

$S / b = (1 \ / 2) (35y - 2 / 3 \ y^2)$

$d (S/b) / dy = (1/ 2)(35 - 4 / 3 \ y)$

$d (S/b) / dy = 0$ ---------- $y = 26.25$ cm

(S/b) max. & q max. @ y = 26.25 cm

q max. = (Q / I) (S / b) max.

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$(S / b)_{max.} = (1 / 2)\,[\,35 \times 26.25 - (\,2 \times 26.25^2 / 3\,)] = 229.6875\ cm^2$

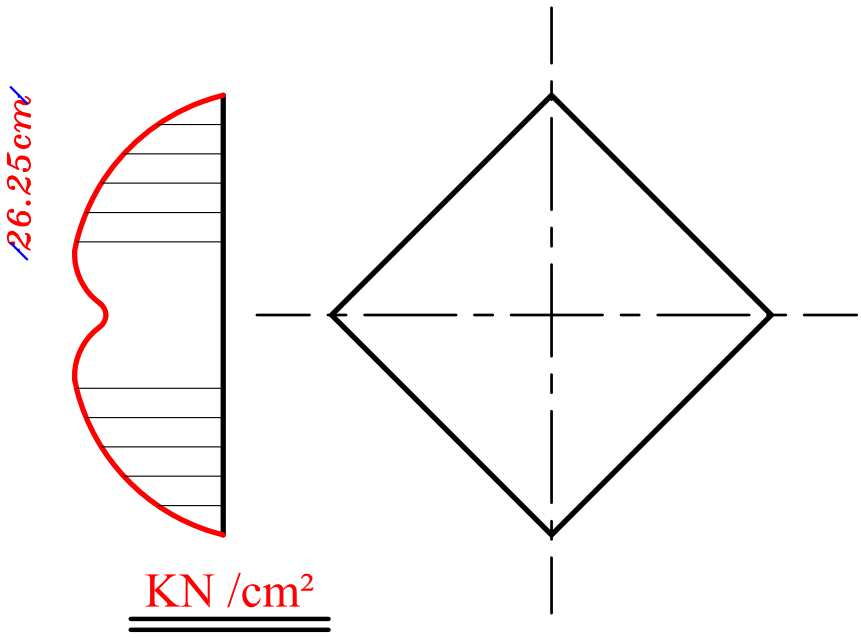
$(S / b)_{c.g.} = (1 / 2)\,[\,35 \times 35 - (\,2 \times 35^2 / 3\,)] = 204.166\ cm^2$

$q_{max.} = (\,500 / 285833\,)\,(\,229.6875\,) = 0.4017\ KN / cm^2$

$q_{c.g.} = (\,500 / 285833\,)\,(\,204.166) = 0.35714\ KN / cm^2$

$q_{max.} = 0.4017\ KN / cm^2$

$q_{c.g.} = 0.35714\ KN / cm^2$



Shear stress distribution (S . S . D)

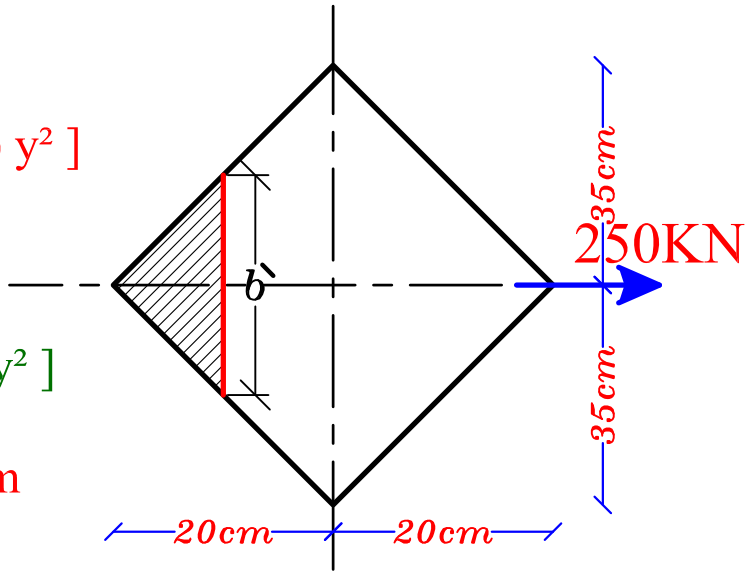
For Q x

$S = [\,(1 / 2)\,(y)\,(b\backslash)\,]\,[\,20 - (2 / 3)\,y^2\,]$

$S / b = (\,1 / 2\,)\,[20\,y - (2 / 3\,)\,y^2\,]$

$d\,(S/b) / dy = (1 / 2)\,[\,20 - (\,4 / 3\,)\,y^2\,]$

$d\,(S/b) / dy = 0$ ---------- $y = 15$ cm



$(S / b)_{max.} = (1 /2)\,[\,(\,20\,)\,(\,15\,) - (\,2 / 3\,) \times 15^2\,] = 150\ cm^2$

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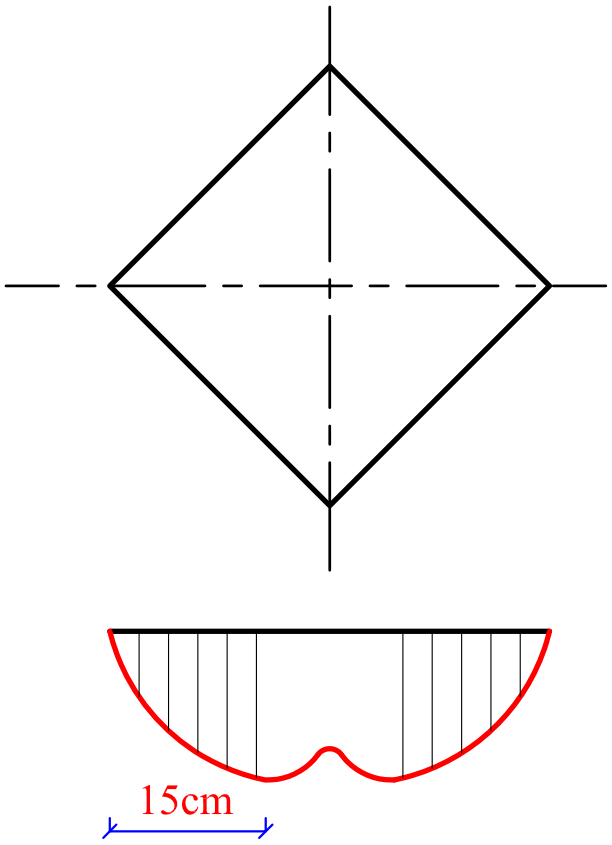
$(S / b)\ max. = (1/2)\ [\ (20)\ (20)\ -\ (2/3)\ x\ 20^2\ ] = 66.66\ cm^2$

$q = (Q / I)\ (S / b)$

$q\ max. = (250 / 93333)\ (150) = 0.40178\ KN / cm^2$

$q\ c.g. = (250 / 93333)\ (66.66) = 0.1785\ KN / cm^2$



$q\ max. = 0.40178\ KN / cm^2$

$q\ c.g. = 0.1785\ KN / cm^2$

Shear stress distribution (S . S . D)

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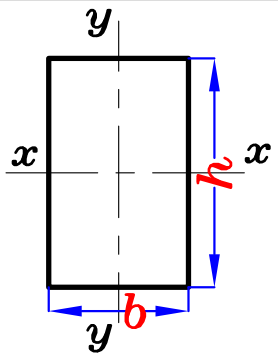
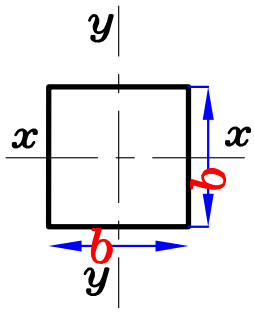
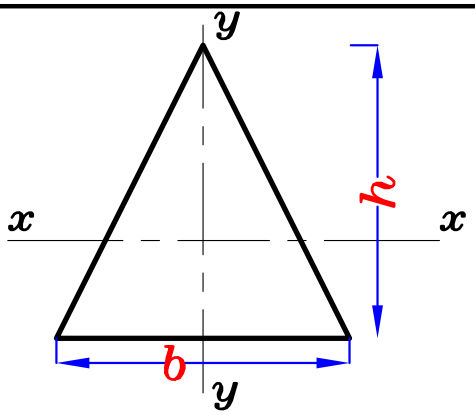
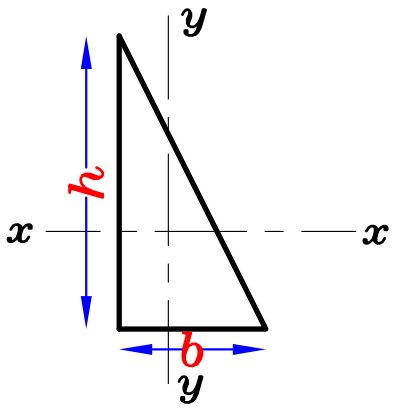
Moment of Inertia

Shape	Moment of Inertia
	$A = b\,h$ $I_x = \frac{b\,h^3}{12}$ $I_y = \frac{h\,b^3}{12}$
	$A = b^2$ $I_x = \frac{b^4}{12}$ $I_y = \frac{b^4}{12}$
	$A = \frac{1}{2}\,b\,h$ $I_x = \frac{b\,h^3}{36}$ $I_y = \frac{h\,b^3}{48}$
	$A = \frac{1}{2}\,b\,h$ $I_x = \frac{b\,h^3}{36}$ $I_y = \frac{h\,b^3}{36}$

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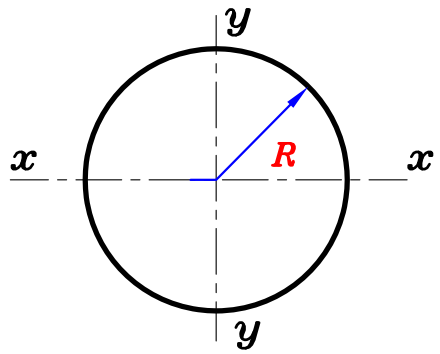
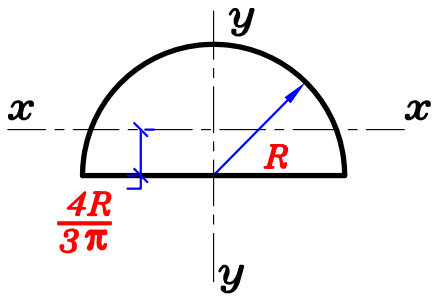
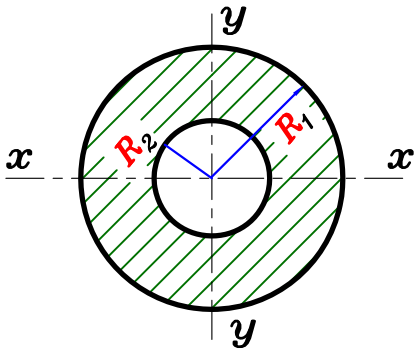
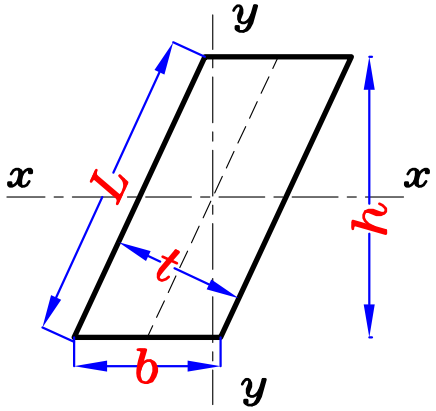
Shape	Moment of Inertia
	$A = \pi R^2 = \frac{\pi D^2}{4}$ $I_x = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$ $I_y = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$
	$A = \frac{\pi R^2}{2}$ $I_x = 0.11 R^4$ $I_y = \frac{\pi R^4}{8}$
	$A = \pi R_1^2 - \pi R_2^2$ $I_x = \frac{\pi}{4}[R_1^4 - R_2^4]$ $I_y = \frac{\pi}{4}[R_1^4 - R_2^4]$
	$A = L\ t$ $I_x = \frac{A h^2}{12}$ $I_x = \frac{A b^2}{12}$

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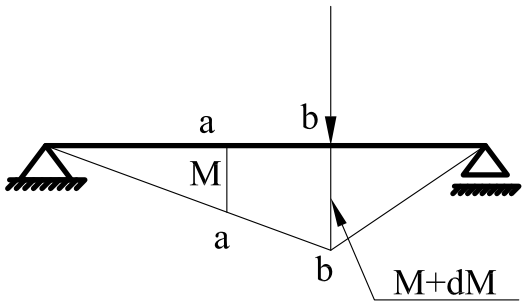
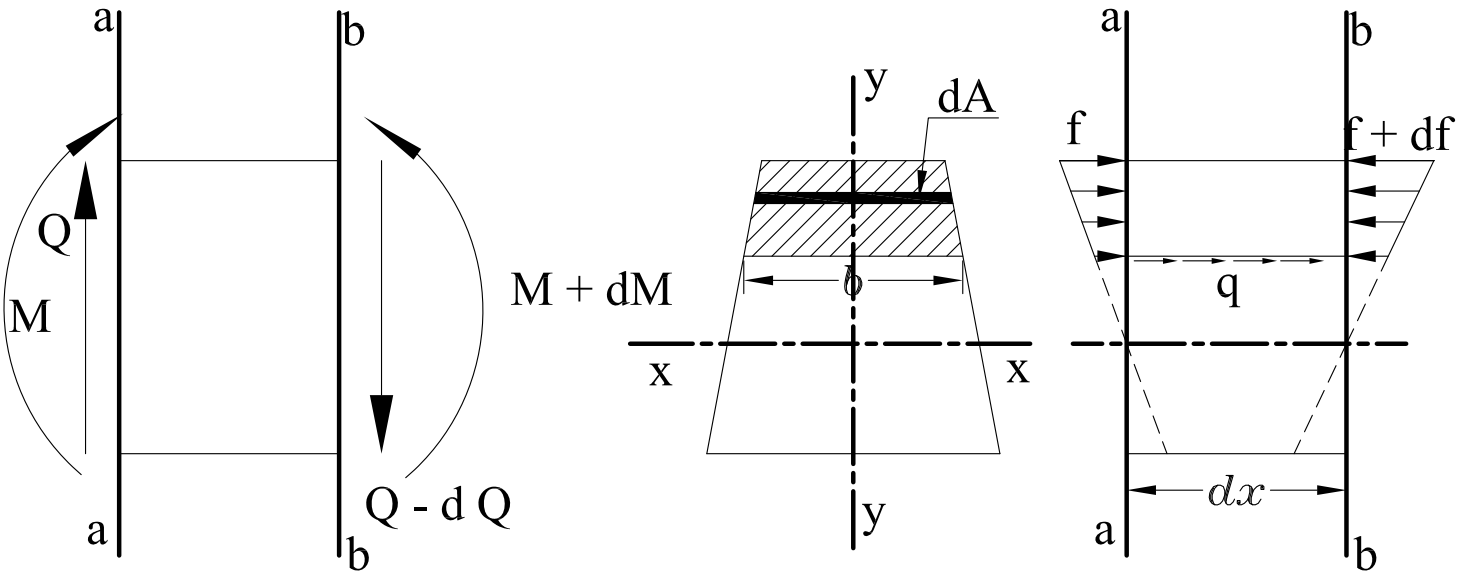
Proof of Shear Stresses

Consider an element of a beam contained between two cross sections a-a & b-b at a small distance apart (dx) and let (M) & (Q) and (M + dM) & (Q +dQ) be the values of the bending moment and shearing forceacross the two sections.

$f = (M / Ix) \times Y$ ---------- face a-a

$f + df = \frac{(M + dM)}{Ix} Y$ ---------- face b-b





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For the Hatched part:

and assumig that the stress q is uniformally distributed along the horizontal plan of area (b dx)

For Equilibrium :

$$q\ b\ dx = \int (f + df)\ dA - \int (F)\ dA = dM / Ix \int y\ dA$$

$$q\ b\ dx = \int [(M + dM) / Ix]\ x\ y\ dA - \int (M / Ix)\ x\ y\ dA$$

$$= dM / Ix \int y\ dA$$

*Where the $\int y$ the statical moment of area (S x)

$$q\ b\ dx = dM / Ix\ (S)$$

$$q\ b = (dM / Ix) / dx\ (S)$$

$$* dM / dx = Q$$

$$q_x = \frac{Q_x}{I_x} x \frac{S_x}{b}$$

$$q_x = \frac{Q_x}{I_x} x \frac{S_x}{b}$$

$$q_y = \frac{Q_y}{I_y} x \frac{S_y}{b}$$

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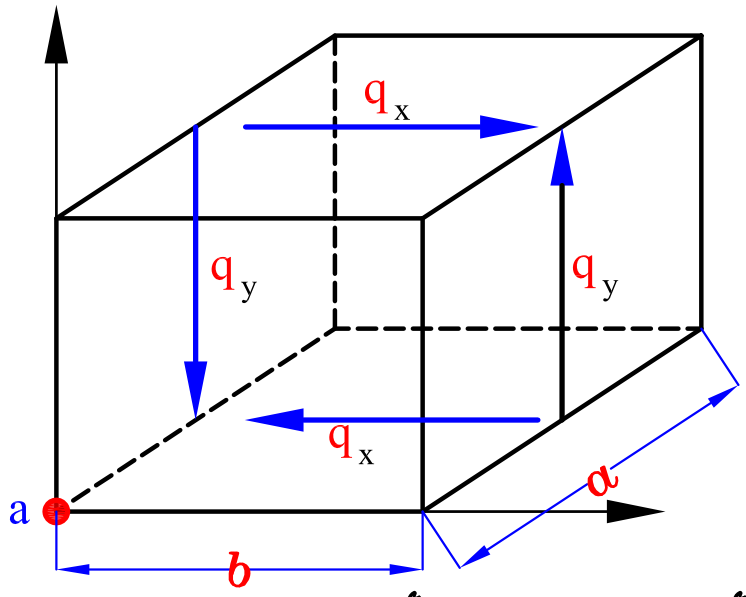
Complementary shear stress:



هذه القاعدة تقول أن أى Shear stress رأسى يكون مصحوب بـ Shear stress أفقى .

فاذا أخذنا مكعب له Shear stress على الجانب الرأسى بمعادلة اتزان بسيطة يتضح أنه نتيجة لهذا الـ Shear stress يتولد Shear stress على السطح الرأسى .

For Equilibrium :

$$(q_y \times a) \times b = (q_x \times b) \times a$$

so

$$q_x = q_y$$

any vertical shear stress is accompanied by horizontal shear stress

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